DOCUMENT RESUME

ED 242 540 SE 044 340

AUTHOR	Wilson, Bryan, Ed.
TITLE	Mathematics Education. Anglo-Soviet Seminar (1st, Oxford, England, September 8-16, 1981).
INSTITUTION	British Council, London (England).; Ministry of Education, Moscow (USSR).
REPORT NC	ISBN-0-901618-64-0
PUB DATE	81
NOTE	140p.
AVAILABLE FROM	British Council, Consultancies Group, 10, Spring Gardens, London SW1A 2BN.
PUB TYPE	Collected Works - Conference Proceedings (021)
EDRS PRICE	MF01/PC06 Plus Postage.
DESCRIPTORS	Calculators; Computers; Curriculum Development; *Educational Change; Educational Planning; Educational Research; Educational Television; *Educational Trends; Elementary Secondary Education; Foreign Countries; Inservice Teacher Education; *Mathematics Curriculum; *Mathematics Education; *Mathematics Instruction; Problem Solving; School Organization
IDENTIFIERS	*England; Mathematics Education Research; *USSR

ABSTRACT

This report presents 23 papers prepared for the first Anglo-Soviet Seminar on Mathematics Education, held in Oxford in September 1981. Ten leading British mathematics educators met with six Soviet participants for an intensive program of information sharing and discussion which covered the teaching and learning of mathematics at all school levels. Each theme of the seminar was introduced by two speakers, one from each country. The speakers highlighted important points in the papers, leaving time for discussion (summarized in the report). The sessions focused on an introduction to the school systems in each country, formative periods, the current scene, the primary stage, the middle stage, research, in-service support for teachers, mathematics for life, the senior stage, and calculators and computers. The closing discussion and a retrospective view of the seminar are included, as well as the program and a list of participants. (MNS)

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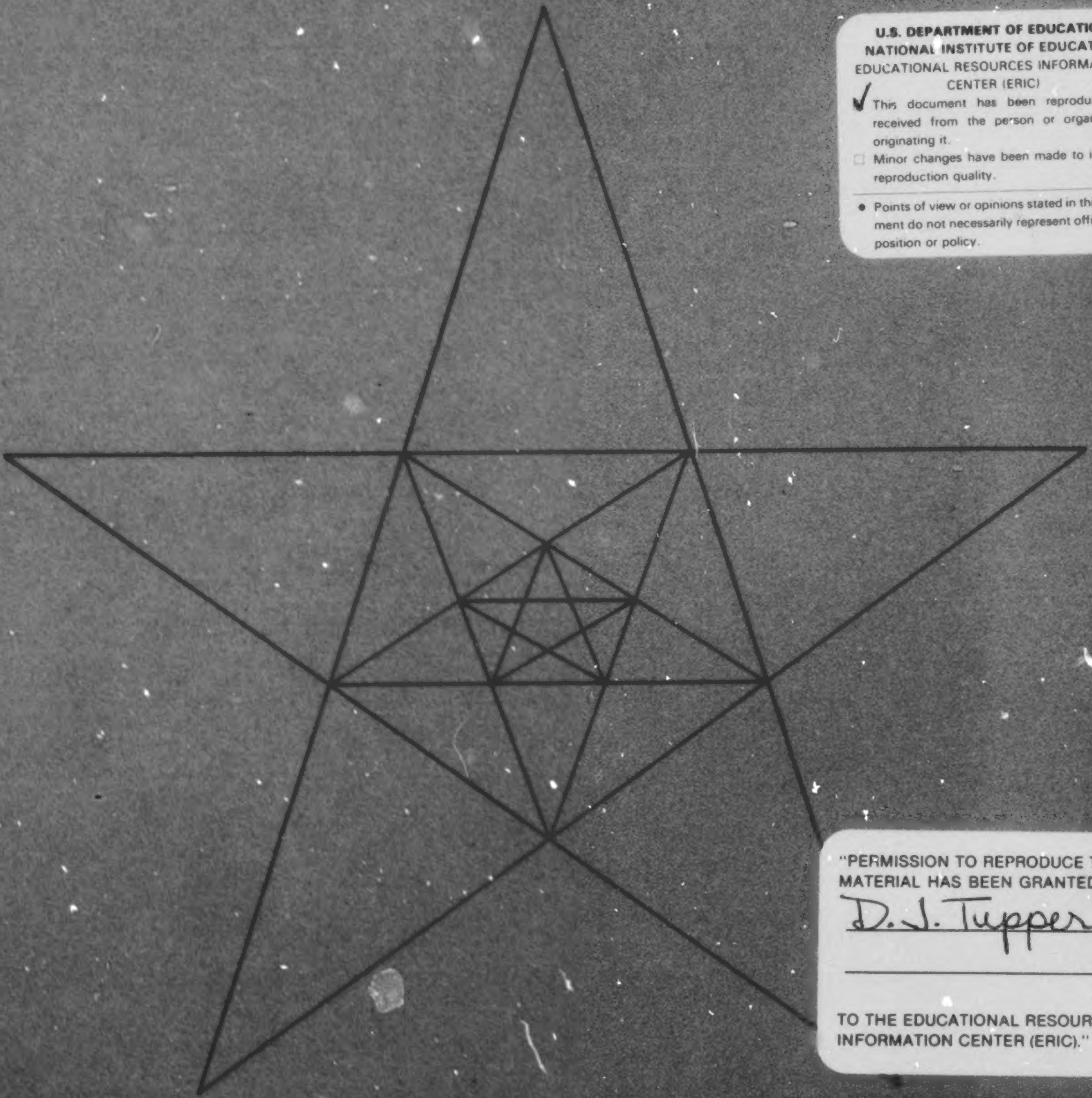

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St Antony's College, Oxford
September 8 - 16, 1981

Mathematics education

the report of the first Anglo-Soviet Seminar
held at St Antony's College, Oxford

September 8 - 16, 1981

under the auspices of the British Council
and the Ministry of Education of the USSR

Chairman Dr T J Fletcher, CBE HMI
Editor Bryan Wilson

ISBN 0 901618 64 0

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PREFACE

The Anglo-Soviet Agreement on Relations in the Science, Educational and Cultural Fields is designed to promote increased contact and greater understanding between specialists in these areas in Britain and the Soviet Union. Under the terms of this agreement, the British Council proposed that a small, high-level seminar be organised to share current experiences in mathematics education at school level in the two countries. It was felt that, although both countries have given much thought to the development of mathematics education in recent years, remarkably little is known in either country about its practice in the other. The invitation was extended by the British Council in the belief that the sharing of experience would be mutually beneficial.

The invitation was readily accepted by the Soviet Ministry of Education, and this seminar was the outcome. Ten leading British mathematics educators met with the six Soviet participants for an intensive programme of information sharing and discussion which covered the teaching and learning of mathematics at all school levels in our two countries.

The seminar programme had a three-part structure. Firstly, we considered the historical development of mathematics teaching in the two countries: how have we got to where we are? We then considered the current situation, looking separately at the primary, middle and senior stages of school education. The final section focused on a few key issues of current concern.

Each theme of the seminar was introduced by two speakers, one from each country. All the papers had been translated and circulated beforehand, thus enabling the speakers to highlight important points, while leaving time for questions and discussion in order to clarify uncertainties and to identify matters of mutual interest. This report reproduces these papers in the order in which they were considered, and summarises the discussions which followed.

Professionally the seminar fully achieved its aims. All the participants expressed their pleasure at how much they had learned. Several key issues were identified as being worthy of further study, and it is very much hoped that a second seminar might be arranged in which some of these could be explored in depth.

Despite the busy-ness of the programme, the British participants were able to show our Soviet guests some glimpses of British culture. All of these were thoroughly enjoyed, whether visiting the Tower of London and Madame Tussauds, punting on the Cherwell, touring St John's College, or watching 'A midsummer night's dream' at Stratford-on-Avon. St Antony's College proved to be a very pleasant base for the seminar; and we are indebted to the College staff who made our stay comfortable.

We also express our thanks to the Headmasters of the two schools to which we had such excellent visits, Mr D J Rule of Sandhills Primary School, Headington, and Mr D H W Grubb of Gillott's School, Henley-on-Thames, and also to Mr T Easterbrook, Mathematics Adviser, Oxfordshire, through whom these visits were arranged.

All the participants were very appreciative of the incisive yet unfailingly courteous chairmanship of Dr Trevor Fletcher, CBE, Staff Inspector of Mathematics, who guided the proceedings with unerring clarity and judgement to and fro across the gulf between two very different educational systems and sets of axioms. The need for everything to proceed by translation could also have proved an insuperable barrier to reaching real understanding; that it did not do so is a tribute to the skill and stamina of the seminar interpreters, Mrs Dorina Offord and Mrs Tatiana Beregovaya. Finally our thanks and appreciation are due to the seminar rapporteur, Miss Daphne Titheridge, whose thoroughness and efficiency behind the scenes ensured that everything ran smoothly.

Bryan Wilson
The British Council

Session I **Introduction**

1a THE SCHOOL SYSTEM OF ENGLAND AND WALES

B J Wilson, Head of Consultancies Group, The British Council

This presentation was based on the showing of a tape/slide programme, with Russian commentary, entitled 'The school system of England and Wales', produced by the British Council. It was designed to supplement a similarly titled article, by Daphne Titheridge of the Consultancies Group, the British Council, which had been translated into Russian for pre-seminar reading by the Soviet participants.

1b THE SOVIET SYSTEM OF EDUCATION

Mr A I Shustov, Deputy Minister of Education, RSFSR

General principles of the public education system

It is the constitutional right of all citizens of the USSR to receive education in their native language, 'irrespective of race, nationality, sex, religion, property and social status.' Educational institutions are publicly owned and provide within a uniform and coeducational system:

- universal compulsory secondary education from age 7-17 years
- vocational, specialised secondary and higher education with work-oriented instruction
- extra-mural, correspondence and evening classes.

Tuition and textbooks are free. Students may also receive maintenance grants and scholarships.

The aim of this system of public education is to provide for the communist education and intellectual and physical development of young people and train them for work and social activity.

The administration of public education

Public education is administered partly by the State (through the Ministries of Education and of Higher and Specialised Secondary Education, and through the Council of Ministers for Vocational Training), partly by Union Republics and at a local level by the Soviets of People's Deputies. Each has its own responsibilities.

The state is responsible for the general principles underlying the provision of education, for drafting all-Union plans for education, for administering its own institutions, for determining the number and types of Union institutions, all staffing quotas, salary rates and student grants, and approving the curricula of all institutions. New proposals are circulated in draft form and widely discussed with the Ministries of the Republics, with teachers and in the press, before final decisions are taken.

The Republics are responsible for drafting republican plans for education, and for administering their own institutions in accordance with existing legislation. The local Soviets administer the institutions under their jurisdiction and take measures to increase their number, locate them and see that provision is adequate.

Structure and organisation of the public education system

For children of 1-7 years there are pre-school establishments - crèches, kindergartens, and kindergarten-nurseries.

Children between 7 and 15 years attend general secondary schools, or schools with occupational training, or schools offering extended study of individual subjects. The very large majority of children enter the next higher grade each year. There is a public examination at the end of the 8th grade. The school-starting age is soon to be reduced from 7 to 6 years of age by the addition of a 'Class 0' to the school system.

Between the ages of 15 and 17, holders of the 8th grade certificate continue in grades 9 and 10, either in general secondary schools (60%), specialised secondary schools, or vocational schools. Others continue their education in evening (shift) or correspondence schools while at work. Non-Russian language schools may continue for another year with the 11th grade. Students leave school with a Secondary Education Certificate and, if they have attended a school with occupational training, a certificate indicating their trade and professional rating.

The school week consists of twenty-four lessons in grade I rising by stages to thirty-two in grade 10. The school year, which begins on September 1st, ends on May 30th for grades 1-7 and a few weeks later for higher grades. From the 7th grade upwards, optional studies are introduced which allow pupils to deepen their knowledge and develop their individual interests and abilities.

Physically and mentally handicapped children are taught first in special kindergartens and then in special schools. Children in poor health attend hospital schools.

Curricula and syllabuses

The curriculum followed in Soviet schools is determined by the state. It includes the humanities, natural sciences and mathematics, manual skills, art, music and physical education. The number of hours per week devoted to each subject is laid down, but varies according to the type of school and stage of education.

The Union Republics draft and approve syllabuses for the first three years, and for the following years in the native language and literature, foreign languages, art and music. The syllabuses in other subjects are approved by the USSR Ministry of Education. They are compiled by scholars in collaboration with leading teachers and the drafts are widely discussed and tested before being introduced. Syllabuses define the content of the subject, theme by theme, and allocate the time to be spent on each.

Teacher training system

Teachers in pre-school establishments, of grades 1-3, and of art, music and physical education, are trained in specialised secondary schools for teacher training. The courses include the theory and practice of education, and further study of special subjects. Teaching practice is given an important place.

Teachers for the higher grades are trained in teacher training institutes for four or five years, and prepared to teach two subjects only. A five-year training is also available in the universities.

On average every teacher upgrades his qualification once in five years by following correspondence courses, or advanced training courses in the universities, advanced training institutes or methods centres. At the present time, teachers of mathematics in the secondary schools attend in-service training courses annually.

Editor's note

A detailed description of the Soviet system of education is to be found in 'Public education in the USSR (1979-1980) and its development', pp. 39-62, published for the 38th Session of the International Conference on Education, convened by Unesco, Moscow, 1981.

1c A FORMATIVE PERIOD IN ENGLISH MATHEMATICAL EDUCATION: 1900-1914

Dr A G Howson, Department of Mathematical Studies, University of Southampton

The period 1900-14 saw developments in mathematics education on an unprecedented scale; indeed, it is doubtful whether any period since has equalled it in interest, particularly within the secondary sector.

Here one must recall that the state provision of elementary education dates from 1870 and that of secondary education from 1902. Prior to those dates schools were charitable foundations or were organised by religious groups, public or private bodies, or individuals. The elementary and secondary systems were, however, still viewed as, to use as a contemporary description, 'two adjacent tenements with an easy passage from near the top of the lower to the mezzanine floor of the higher'. The passage was, in fact, far from easy - even by 1911 only one in twenty-three elementary-school children negotiated it. It was a significant bridge to cross, for it took one from a sector of education designed for the working classes and staffed by poorly paid, often ill-trained 'teachers', to a primarily middle-class institution having better paid, better qualified staff who enjoyed a higher social status.

Elementary schools

There were significant changes within the elementary schools during the period under consideration, but these were more in official attitudes than in classroom practice. Until the 1890s the financing of elementary schools had been partly dependent upon the results obtained by their pupils when examined on a state-prescribed syllabus. This had ensured considerable uniformity of method and of content. When 'Payment by Results' came to an end there was an attempt to encourage teachers to use their new freedom. Thus a Board of Education handbook published in 1905 stated that 'the only unanimity of practice which the Board.... desires to see in Public Elementary Schools is that each teacher shall think for himself, and work out for himself such methods of teaching as may use his powers to the best advantage and be best suited to the particular needs and conditions of the school'. It thus helped to give added credence to that great 'myth' of English education: the autonomy of the individual teacher.

Yet how could a teacher who had never been trained to think, and who knew little of the range of possible methods of teaching, act upon such advice? It was forgotten that autonomy can only be exercised once understanding and experience have been gained. Schools continued to use the syllabuses and books they had used for years and arithmetic as prescribed in the 1890s continued to be almost the only mathematics taught. Any attempts to break free from the stranglehold in the early years of schooling were also thwarted by the pressures of the 11+ examination, success in which enabled the pupil to transfer to the secondary school. This examination placed great emphasis on 'sums'.

There were, of course, individual innovators at that time working within the 5-11 age range. For example, Mrs Boole urged the need for pre-mathematics and the cultivation of geometric imagination. Her curve-sewing cards achieved some popularity but were unlikely to have been found in many state elementary schools. It is also doubtful if the 'play-methods' she advocated for pre-mathematical ends were ever used as a foundation for later mathematical work. (The same criticism could be levelled at more recent experiments.) Apparatus was also being used in some schools, in particular the abacus and Tillich's bricks (precursors of Cuisenaire rods). However, the general results in the 5-7 age range were such as to cause the senior mathematics inspector in London to suspect that specific lessons in number to children under the age of seven served no useful purpose.

The general movement towards practical work was greatly hindered by large class sizes, often fifty or sixty pupils, and this also resulted in too much written work and too little oral. These problems were recognised by mathematics educators, but in the prevailing conditions little could be done to counter them.

The abolition of 'Payments by Results' had meant that less emphasis was given to arithmetic within the curriculum as a whole and that arithmetic teaching did not concentrate quite so much as previously on the practice of sums. Not surprisingly, there were soon complaints that accuracy was suffering and that new methods were to blame.

Central schools

One development within the elementary sector of particular interest was the emergence of 'central schools'. These were intended for the brighter, urban children who had not been selected for grammar schools and who were able to attend school beyond the age of compulsory attendance. (At that time this varied within the country. The age was set locally at either 13 or 14 but over-12s were able to obtain exemption from attendance either partially or wholly.) The curriculum had a marked commercial and/or industrial bias and was often varied to suit local needs. For example, 'the course will aim at giving the power of accurate and rapid calculation with manageable numbers; abundant exercise on well-chosen concrete material, including both indoor and outdoor measurements thereon; ability to deal in an independent and commonsense way with any simple problem that may present itself; ability to read and use any workshop formulae; good scale drawing, including a simple rough sketch from machine details or from observation or description, and finished drawings to scale; as much construction work in plane geometry as is necessary for handicraft, mechanical drawing, and general workshop demands; some experimental geometry leading to a habit of investigation and a knowledge of the simple properties of the common geometrical figures and solids and an introduction to the Euclidean method of reasoning; some knowledge of literal expression and the application of algebra to the solution of problems; a knowledge of the trigonometrical ratios and ability to calculate heights and distances; daily practice in the use of logarithmic tables in calculations; the ability to construct and read a simple graph; the ability to use profitably a good mathematical class text-book'. Emphasis was also laid on the use of such instruments as the slide-rule, vernier, micrometer and theodolite.

It was a form of mathematics education later to be found in the technical schools, but which has now disappeared, having been replaced by courses which have a greater academic and less vocational bias.

Education at the elementary level, then, was making steady progress in the period under consideration. The lessons to be learned from a study of it lie chiefly in the domain of curriculum development, and concern the degree of autonomy which teachers can sensibly be called upon to exercise, and the problems of persuading/getting ill-trained teachers working under difficult conditions to adopt new methods.

One present-day educational problem, the differentiation of curricula, was 'solved' in an obvious way: an academic curriculum within the secondary sector (for future 'white-collar' workers), an advanced vocational one within the central schools (for the future artisan), and a 'social' (day-to-day living) curriculum in the ordinary elementary schools (for the common worker). Since middle-class children had automatic entry to the secondary schools there were few articulate opponents of this system. (An influential Labour Party policy document 'Secondary education for all' was to appear in 1922 (edited by R H Tawney). This demanded an end of the 'two tenement' approach and asked for a unified primary-secondary system for all children 'irrespective of the income, class or occupation of their parents'. However, a variety of secondary education was visualised: there was 'no wish to sacrifice the peculiar excellence of particular institutions /the grammar schools/ to a pedantic State-imposed uniformity', 'not all children respond equally to the same methods and curriculum. Equality of educational provision is not identity of educational provision, and it is important that there should be the greatest possible diversity of type among secondary schools'.)

The period 1900-1912 was to see secondary education become more uniform. In the second half of the nineteenth century the government had established a Department of Science and Art (DSA) intended to promote scientific and technological education. One way in which it did this was by providing grants to schools whose pupils took its examinations. These had a strong scientific bias and the mathematics was vocationally inclined. For example, there was considerable emphasis on practical geometry (at a time when a grammar school pupil might never see a protractor). In the closing years of the century, the DSA introduced a 'practical mathematics' syllabus designed by John Perry, an engineer, ex-foundry-apprentice, ex-schoolmaster, and at that time a professor in higher education. The DSA grants attracted many schools to take its examinations, both local authority 'higher-grade' schools and also the poorer grammar schools. The richer grammar schools tended, however, to look down on its technically-biased curriculum.

In 1902 an Act of Parliament created a state secondary system which was initially formed from existing schools, with the DSA-type schools in the majority. However, liberal educators complained of the curriculum followed in these schools and in 1904 a uniform curriculum was imposed nearer in spirit to that of the old grammar schools. The conflict between the old academic tradition and the newer developments also arose within mathematics education. For years many teachers in the grammar schools had wished to teach geometry in a form different from Euclid. However, they were prevented from doing so because Cambridge University - the great mathematical power in the land - refused to change its examination requirements. An association of teachers (the Mathematical Association) had been formed to effect changes but by 1900, after thirty years of unavailing effort, it was dispirited. At this time a new force appeared on the scene: Perry demanded that all of school mathematics should be radically overhauled and, in particular, that it should be redesigned to serve the needs of utility. It should be aimed more at producing practical men, scientists and engineers, than pure mathematicians, a view shared by many of the country's leading scientists and engineers.

The clamour was so great that Forsyth, the leading Cambridge professor, was forced into action. Perry's demands were anathema to many academic mathematicians and a more acceptable reform programme had to be found. Forsyth turned to Godfrey, a young teacher in an independent school, and he, with the help of colleagues, produced a less radical set of proposals. Nevertheless, Godfrey too was to emphasise the utilitarian aspects of mathematics. Until the 1890s mathematics had secured a place in a liberal curriculum because it acted as a 'mental discipline' and helped develop certain desirable 'faculties'. Suddenly, however, 'faculty' psychology was undermined; there seemed to be little evidence of transfer of training. Herbartian psychology became influential in England. This, however, proved more valuable in indicating how one might teach than what one ought to teach. Godrey responded to its emphasis on appealing to the interests of the pupil and interpreted this as pointing to the need to bias mathematics teaching towards utility. Thus, possibly for different reasons, Godfrey and Perry had many shared objectives.

The creation of the new state secondary system, the debate begun by Perry, and the belated realisation by Cambridge that it must do something, brought about a revolutionary atmosphere in secondary school mathematics. Godfrey compared it to the break up of an ice-age. The changes and some of the problems they raised can be summarised as follows:

1. Content

a. Geometry

Although the overthrow of Euclid was a great symbolic step, it was not perhaps the most important curricular change. A number of new contenders arose: practical (with little deduction), neo-Euclidean (a reordering of Euclid with new proofs supplied), and compromise (practical preliminary work followed by deductive chains of theorems). It was the last which was to prove most successful in publishing terms. However, in the classroom the preliminary practical work was often either ignored or treated as a task to be completed once and for all before getting down to the serious business of geometry. Examining the 'new' geometry proved difficult and there were calls for a nationally agreed sequence of theorems. How to examine geometry successfully has proved a perennial problem and the resulting backwash on geometry teaching has been far from beneficial. (Godfrey's views on geometry teaching changed considerably in the following two decades, but it was his first book of 1903 which continued to sell (into the 1970s) rather than his later ones.)

b. Other topics

A questionnaire answered by 370 secondary (grammar) schools (independent and state) about 1911 showed that about half undertook out-of-doors practical work in surveying and mensuration using such instruments as the theodolite and the plane table. Descriptive geometry (in the sense of Monge) was taught in over half the schools. The diagrammatic representation of data and other elementary statistics were fairly common. The graphical representation of functions was now taught in almost all schools (squared paper had become increasingly available and cheap) and, somewhat more surprisingly, about half the schools claimed to use vectors in connection with the teaching of mechanics and/or complex numbers. Graphical methods were used in statics and 4-figure logarithmic tables were now used in preference to 7-figure ones. Trigonometry was now securely established in the school curriculum and calculus was taught more and more frequently to 16 to 18 year olds of average ability and even to pupils under 16.

These changes in general proved permanent, although practical work tended to go out of fashion along with descriptive geometry.

2. Method

Perry had put much emphasis on a practical, laboratory-based approach to mathematics and Godfrey, too, was sympathetic to this. His and other leading independent schools opened mathematical laboratories and much emphasis was placed on measuring activities and on mechanical experiments. It was a movement that did not survive long. Not every school possessed such a laboratory and in those that did there was a reaction in favour of more academic work. The laboratories tended to be handed over to the physicists and the feared dichotomy - theoretical work to the mathematician, and practical work to the physicist - came to pass.

In general the hopes of those who battled for more practical work were dashed. Some found its way into the curriculum, but not as much as had been hoped and the role it played in the teacher's conception of the curriculum was not always what was originally intended.

3. New movements in mathematics

A Cambridge college in the late 1800s set the examination question 'define a function and prove that all functions are differentiable'. Soon, however, Jordan's 'Cours d'analyse' had prompted English equivalents and such questions perished. Cambridge mathematics took a new turn and analysis became prominent. In 1906 the Cambridge Mathematical Tripos examination was 'reformed'. At the time, this was hailed as an enormous step forward putting emphasis on understanding rather than the ability to solve stock examples and tricky problems. In retrospect, the changes did little to weaken the influence of examinations on the university (and school) curriculum. The change to rigorous analysis and the widening gap between how the calculus was approached at school and at university was perhaps more significant.

4. (Temporarily) lost causes

Not all the ideas advanced in the period under discussion were sympathetically received. Many were, however, to re-emerge years later. Thus, W J Dobbs produced a book in which plane geometry was studied via the elementary notions of 'rotation', 'translation' and 'folding'. Boys, he claimed, were brought up 'to believe that the subject of congruent triangles is at the root of all Geometry ... instead of being ... of relatively small importance' - a Dieudonné of the 1910s! His book proved as unsuccessful as that of David Mair (1907) who produced a unified mathematics textbook in which the different strands of mathematics were interwoven, and which used the problem as motivation. Its like was not to be seen again until 'SMP book 1' in 1964. Carson argued that using '-' to denote 'subtract' and 'negative' led to confusion and proposed a distinct notation for negative numbers, '5'; again, an idea to be taken up anew in the SMP series in the 1960s.

Perhaps these 'failures' reflect the inability of the educational system to digest more than a limited number of changes at any one time. It is interesting to observe though that those who revived these and other ideas in the 1960s were largely ignorant of their predecessors' work: it was solely that a similar background and ideology led them to suggest similar remedies to persistent problems.

Yet another temporarily lost cause was the Association of Teachers of Mathematics (for the South-Eastern part of England). Carson thought that the Mathematical Association concentrated too much on the concerns of the grammar school and that its periodical, the 'Mathematical gazette', could not be read by the average teacher. He formed a new association in 1911. Originally it had forty-six members - the majority being women! Its 'Journal' was to appear only eight times before war broke out and the association's activities ended. The new association's creation, and, indeed, the significant growth in membership of, and the increased level of activity within, the existing Mathematical Association indicate the need felt by teachers for co-operative support, in a period when they were subject to new stresses.

5. The education of girls

Girls' secondary education dated effectively from the second half of the nineteenth century. A government commission in the 1860s found that in the whole of the country less than 650 girls were attending endowed secondary schools. Subsequent attempts to extend secondary education for girls proved, however, very successful. The curriculum in the girls' schools came to be determined by the syllabuses of the examination boards which were drawn up with boys in mind. For many years pioneers of women's education sought to gain recognition by fighting on the existing academic terms, which meant accepting the boys' curriculum. However, in the period under discussion serious questions were raised about the place of mathematics in the education of girls. Could it have attractions for girls and could the average girl succeed at it? (The girls' schools were bedevilled by a shortage of qualified mathematics and science teachers.) What were the consequences of the switch from teaching mathematics as a 'mental discipline' (now partially discredited) to teaching it for 'utility'? Was it not now geared solely to the production of scientists and engineers, or, put another way, was the 'new mathematics' only for boys?

Unfortunately, women teachers were not represented when decisions were made about the new syllabuses, neither did any participate in the writing of the widely-adopted texts. The special needs of girls were ignored (or the possibility that they might exist was not discussed). The problems remained.

6. Curriculum development

This period of frantic change first laid bare many of the problems which confront curriculum developers. Material was received into classrooms, but the spirit which had motivated it was not comprehended. The teaching force lacked the necessary understanding and qualifications - as the then chief inspector pointed out, lack of qualifications can be overcome in an individual or a small group, but not if it affects the main body of teachers. Changes, too, percolated to schools in a haphazard fashion; many schools went on 'teaching in the old way'; the intention of the reform 'was quickly forgotten or not understood'; an absence of 'an accepted guiding principle' led to 'various fantastic and ill-balanced developments'; textbooks were adhered to 'slavishly'; and the limited in-service education that could be provided (mainly lectures) was woefully inadequate. That changes in content are more readily absorbed than changes in method was clearly demonstrated. Mathematics education suffers from a regression to the blackboard and the academic.

7. Teacher training

It was accepted by 1840 that there were benefits to be gained from providing elementary school teachers with special professional training. Not until 1879 was it recognised that grammar school masters might also benefit from this. In that year Cambridge organised three lecture series, on the history and the psychology of education and on how to teach. (It was, as a 1921 author put it, equivalent to teaching anyone to play cricket by lecturing them on the techniques of the game and on the lives of great cricketers.) Soon, however, the professional training of secondary school teachers came to be linked with the universities, which opened departments of education and ensured (in a way that many countries still do not) that professional training included a substantial amount of school-based work. By 1911 Percy Nunn could claim that it was becoming generally accepted that all teachers should have professional training. (He was over-optimistic; mathematics graduates are still not required to have been trained.)

8. International links

The establishment of the CIEM (ICMI) in 1908 brought England's mathematics educators into closer contact with those in the USA and Europe. Increased interest was taken in conditions and curricula elsewhere and many ideas were imported, for example, on the use of intuition and the teaching of functionality from Germany, and on the use of symmetry in geometry teaching from France. These new links offered much and the 1912 surveys produced by CIEM stimulated considerable thought. International co-operation was, however, to come to an abrupt end in 1914.

9. Laying the foundations of a discipline of mathematics education

Teachers in the nineteenth century had been largely content to function as teachers and to make such curriculum changes as they felt desirable in a pragmatic way. Books were written on 'the philosophy of education' and De Morgan published articles on the specific problems of teaching mathematics, but it was not until the first decade of this century that significant attempts were made to study mathematics education rather than to practise it. Here, again, the work of Klein, D E Smith and others abroad may have proved a stimulus, but suddenly, about 1907, mathematics education began to blossom. Godfrey wrote articles for the 'Gazette' which were innovative for that journal, emphasising educational principles and theories rather than mathematical detail. The following year saw the publication of Branford's 'A study of mathematical education', a remarkable, original work. Less encyclopaedic, but equally worthy of respect, was Carson's 'Essays on mathematical education' (1913). Nunn's three-volume analysis of the teaching of algebra appeared in 1914, and exemplified the serious approach to mathematics education established in the earlier decade.

The period from 1900 to 1914 was, then, a most fruitful one for mathematics education in England. There was, moreover, no indication in 1914 that the period was about to end, but end it did. Whether it was the loss of a generation of young schoolteachers who would carry on from where Branford, Carson and Godfrey had led, or whether the system needed a period of consolidation in which to digest the many reforms, is not clear. Regrettably, however, the impetus for change was lost, and was not to be restored for many years, and significantly the quality of work within the field of mathematics education was to drop.

In his presentation, Dr Howson selected some topics from his paper that he thought might provoke further discussion:

1. What are the appropriate age and methods for moving from pre-mathematics, with an emphasis on activities and oral work, to more formal written mathematics?
2. The influence of academic criteria on the syllabuses now used in both countries for comprehensive schooling has been overwhelming.
3. When belief in the transfer of learning was shattered, people had to rethink reasons for teaching. On what theories should we plan a curriculum now?
4. Too little is known about the spiral curriculum. What is the best size for topics? How long should we spend on one topic? How frequently should we return to it - within the same year or in another?
5. The mathematics syllabus is unified in the USSR up to the 6th grade and throughout in the UK. Is mathematics in fact as unified a subject as it is taught? How can its unity be demonstrated? Should its division be algebra, geometry and arithmetic - or maybe finite and infinite mathematics - or something else?
6. What is the purpose, and the optimum nature and quantity, of practical work in the learning of mathematics at various ages?

Several of these topics - the spiral curriculum, the unity of mathematics, and the balance between pre-mathematical activities and formal mathematics - were discussed at this point, and taken up again later in the week. The balance between theoretical and applied mathematics was also suggested as a topic of concern in both countries.

2a REVISION OF THE CONTENT OF MATHEMATICS EDUCATION IN THE SCHOOLS OF THE USSR DURING THE 1960s AND 1970s

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In the years 1960-70 in the USSR good work was done in including new subject matter in the school mathematics course, and in improving the methods of teaching following the institution of universal secondary education for significantly increasing numbers of pupils.

During this period an intensive study of ways of further development of the school mathematics course was conducted. This is entirely natural. The scientific and technical revolution speeds the pace of development in science, technology, economics, culture; it raises the role of intellectual work in all spheres of man's activity; its level and further development is fixed to a great extent by the educational potential of the country, which is directly linked with the improvement of schools and general education.

It is natural that with scientific, technical and social progress the aims of education, the subject matter, and the methods of teaching change. Something ages; new questions arise, which are included in the circle of knowledge and skill which is essential for all pupils; demands on the quality of personality which must be developed in the pupils are more accurately defined. All this leads to the neccessity of systematically analysing the conformity of the results of teaching mathematics with the aim of mathematical education, and brings corresponding corrections into the subject matter and the method of teaching. In this way the improvement of school education is a continuous process.

In the process of improving school mathematics education one has to resolve a series of objective contradictions of a dialectical character. These are exemplified by the contradiction between the growing volume of knowledge the mastery of which is desirable for all pupils, and the limitation of school time; the contradiction between the necessity of raising the theoretical level of the mathematics course and the realisation of universal secondary education, which makes necessary an improvement in the accessibility of material. The improvement of the effectiveness of teaching demands the perfection and modernisation of the teacher's methods of work, but this is made difficult by the fact that many teachers received their training in circumstances before the present demands for a mathematical education were in existence. There is also a contradiction between teaching everyone by a single curriculum and the necessity of taking stock within this framework of the different interests and aptitudes of pupils.

The realisation of the fundamental direction of improving school mathematics education is defined by its methodological basis, its aims, and the problem of schools of general education.

The methodological basis of school education in socialist countries is the Marxist-Leninist concept of the mutual relations between scientific knowledge, the world view, and practice in the complex transition to a communist upbringing.

The teaching of mathematics offers help to the solution of these problems. It must guarantee a stable and conscious mastery of the basis of mathematical knowledge, of skills and practices necessary for the study to a sufficiently high level of the course of mathematics itself (and of related disciplines), for the mastery of professions, for the continuation of education in higher and middle specialist educational establishments, and for self education. An understanding of the role of mathematics, of its methods, of the relationship of mathematics and practice, becomes an integral part of the common culture of man. The teaching of mathematics must help the formation of qualities which guarantee the sufficiently quick adaptation of youth to the ceaselessly changing conditions of activity in the NTP* epoch. In connection with these, in the school course of mathematics not only must the demands of the present day be reflected, but as far as possible also the prospects of change in the demands made by society on school mathematics education.

An analysis of scientific, technical and social progress enables us to draw the conclusion that in the improvement of the school mathematics course there must be certain objective general progressive tendencies in its development. These are, firstly, the strengthening of the universal educational role of the mathematics course, its humanisation, secondly, the strengthening of the applied and polytechnical direction of the course, and finally, the improvement of its theoretical level.

National and international experience of modernisation of school mathematics education in the last decades confirms the correctness of the above statement. However the existence and character of the reflection of the tendencies in school mathematics courses is far from uniform and is defined by a series of factors. The most important of these is the social and political structure of society, its aims and the system of school education (mathematical in particular), the academic level achieved (including pedagogics, psychology, and the methodics of mathematics) and the technical, cultural and economic level.

The complex appearance of these tendencies in socialist countries depends, first of all, on the aims of school education in a society of developing socialism in the conditions of NTP: the communist upbringing of young people, the harmonious development of personality, the understanding of the role of mathematics in the development of science, technology, production processes, economics and culture. The complex calculation of these tendencies extends the possibilities of improving the general educational potential of society, and strengthens the pupils' motivation for learning. A sufficiently full reflection of these tendencies in contemporary courses of mathematics became possible through the connection with the latest achievements in the fields of child psychology, pedagogics and the methodics of mathematics.

Breaches of equilibrium between these tendencies in the development of the school course of mathematics lead to the necessary correction in curricula, and to the rewriting of textbooks.

General tendencies in the development of the school course in mathematics are shown in the systematic renewal of its content, in the improvement of the structure of the course, and in the direction of teaching. They receive a corresponding reflection in the teaching methodics provision of the course, in the curriculum, in textbooks, in didactic material for pupils, in methodic appliances for the course, in a system of teaching aids, and in the maintenance of teacher training.

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*NTP is an abbreviation for 'Scientific and Technological Revolution'

For the contemporary school course of mathematics there was a characteristic systematic renewal of its contents in conformity with the aims of teaching and the purpose of its development. One of the principles consistently realised in the examination of contents and methods of teaching in socialist countries is a careful attitude to the experience of school work, preserving a sufficiently large stable nucleus (number, expression, algebraic manipulation, equations and inequalities, function, elements of synthetic geometry); traditionally, great attention is paid to the solution of written problems. The strengthening of the course on functions and the role of geometric transformation, containing elements of analysis and vector algebra, is an extension of this stable nucleus. In view of the strength of the theoretical and applied significance of these questions they will doubtless keep their place in subsequent curriculum reviews.

We note that such an approach - a careful attitude to the experience of school work - is far from being accepted by all. Many extreme modernists consider it necessary to build entirely new courses ('Calculation is not mathematics'; 'Euclid must go') in which the intensification of a formal basis plays an important role. It is entirely natural that these attempts are not crowned with success.

The renewal of the content of the course of mathematics has, in comparison with other subjects, its own special character. For instance it is to be observed that in the physics and chemistry courses the effort to reflect to some measure the achievements of contemporary science is sufficiently clear. In view of the complexity of mathematical equipment and the problems solved by modern mathematics, the question of the school mathematics course is on a much more modest scale. We can only talk of the mastery of sufficiently simple ideas, comprehension, methods widely used both in mathematics and in its applied forms, independent of the stage of the development of science at which they appear. And only separate information which is related to contemporary mathematics (sets, algorithms, and understanding of computer programming) enters the school mathematics course, necessarily in a very simplified form, and is, on principle, seen as being for the purpose of getting to know about it.

Together with the renewal of the school mathematics course, its content and terminology, there is also the exclusion of old material, material which has lost its educational value (theory of operations on the slide rule, formulae for the volume of truncated pyramids and truncated cones), enriched by new methods , the rejection of the old (for instance, the study of the function by elementary means gives place to the study of function with the help of the derivative, and so on). Due to the greater number of pupils the exact apportionment of demands for knowledge and ability in pupils acquires special significance.

In academic curricula and textbooks there is clearly to be seen an effort to reflect in the structure of the course the unity of mathematics, of its understanding and methods. This effort is the result of two circumstances: the outer, the deep connection between different branches of mathematical knowledge, and the inner, pedagogic searches for an expedient system for mastery of mathematical knowledge by pupils.

The division of the mathematics course, which has developed historically in the Soviet Union into separate disciplines, creates particular complications in the organisation of academic material and the process of teaching itself; the strengthening of the inner connections cannot eliminate the inevitable doubling of material. Already at the present time work has been begun on integrating the mathematics course. Such courses already exist in a number of socialist countries. A unified course of mathematics is being studied in Estonian schools, where a large scale experiment in the creation of a unified mathematics course has begun.

Methodical principles of building a mathematics course are directed towards making the course more accessible, of creating in the pupils a correct understanding of the place of mathematics in the life of society, and on the development of the thinking of pupils.

We cite the most important of these:

1. The grouping of all material around a compact system of pivotal (support) understanding, which makes possible the formation of a conception of mathematics as a united science, allowing the establishment of links between concepts formerly taught in isolation, and thus making easier the mastery of the material.
2. The formation of the most important concepts, skills and practices in the course of a protracted period.

So, for example, the understanding of function (mapping) is one of the pivotal concepts of school mathematics. It is first introduced into the 6th class, but preparatory work for its introduction begins significantly earlier. Already in the 4th and 5th classes pupils find the values taken by variables over a set of points, they become acquainted with concepts of the co-ordinates of points on a straight line and in a plane, they do exercises in the construction of diagrams and the reading and constructions of graphs, and they study elementary ideas of the properties of transformations. All these questions are extremely important from the point of view of the training of pupils in the perception of the idea of function.

The understanding of function, formed in algebra and geometry in the 6th class, is improved and supplemented in the 7th and 8th classes, where the idea of function finds significantly greater use than in previous programmes. Questions such as numerical sequences and geometric transformations are treated from the functional point of view.

The study of function is conducted is a qualitatively new way in the 9th and 10th classes, where an understanding of derivative and integral is introduced and transformation of space is examined (vector, axial and central symmetry, symmetry in two dimensions).

Similar is the matter of the learning of algorithms. For instance, the learning of calculation algorithms is established during the whole eight years of school.

3. The apportionment of different levels of material. Formerly the curricula and textbooks in use envisaged the same manner of mastering material, the same knowledge of all the appointed material that was in the textbooks, and the proofs of all theorems. Furthermore sometimes the proofs were introduced without clear indication of logical omissions in the proofs.

The course of mathematics studied in school at the present time is constructed differently. Definitions are made which the pupil must learn by heart. The reproduction of certain other definitions, formulated in the textbook, is not required.

Analogous with this is the matter of theorems: in textbooks a set number of theorems are separated, the proof of which pupils should be able to reproduce. For some theorems, proved in the course, the reproduction of the proof by the pupils is not obligatory. A number of theorems are postulated, although only the possibility of proving them is indicated.

This approach makes it possible to reduce the general load on pupils, which is extremely important in the conditions of transition to universal secondary education and the increase in the number of pupils in school.

We note that in the textbooks material is included for further reading, in particular historical information and problems of greater difficulty. In this way conditions are created for satisfying the interests of pupils showing a deeper desire to study mathematics.

4. The gradual intensification of the role of the deductive method. In the 4th and 5th classes the inductive exposition of material predominates. Concepts, laws, facts are first of all explained by concrete examples and only after this are generalisations made. This applies as much to material traditionally taught in these classes as to material taken from senior classes. For instance, basing themselves on concrete examples, pupils from the 4th class formulate the criterion of divisibility for 2, 3, 5 and 10. But already in these classes exercises are done to foster in pupils the need to prove (though the word proof is not used at this stage). The first examples of deduction appear. So, by the inductive path, on the basis of generalisation from the pupils' experience of life, an idea of the parallelness of straight lines is formulated. Applying this idea, the parallelness of the opposite sides of a square and of a rectangle already have a deductive character. Such a combination of inductive and deductive approaches is usually used for the construction of the axis of symmetry of two points, for the teaching of the laws of operation of rational numbers, etc.

5. A consistent and single minded realisation of the connections within subjects and between subjects is expressed in the timely introduction of concepts and the forming of mathematical equipment necessary for the study of mathematics itself and of contiguous subjects. This approach enables the teaching of allied subjects to be undertaken on a sufficiently high theoretical level, systematically shows the pupils the power of mathematical methods, works continually for the improvement of the level of knowledge, skill and practice, in fact increases the time for their formation.

6. The aspiration to guarantee the accessibility of material and its mastery by all pupils, the calculation of the psychological particularities of mastering the material and the changes of content of the courses have led to a broader understanding of visual aids. Formerly in mathematics textbooks much attention was given to drawings, diagrams and sketches. However here a new qualitative approach to illustration has appeared. Draughtsmen have started to play a more active role in the formation of concepts than formerly. Fairly simple visual models began to occupy a large place, effectively revealing the characteristic trait of the concept being taught. Thus the introduction and use of co-ordinates on a

line and co-ordinates in a plane already in the 5th class make possible presentations of algebraic relations with the help of geometric images and thus significantly ease the mastery of many questions by pupils: comparison of numbers, operations over positive and negative numbers, revealing the meaning of the concept of modulus of a number, solving equations, inequalities and their systems. In senior classes a co-ordinate straight line finds wide use as a visual aid for forming an understanding of the environment, a modulus of difference of numbers x and a (as the distance in the co-ordinate straight line between points x and a) etc.

7. The broadening of the function of exercise is expressed in an active use of exercises in all stages of teaching: in preparation, introduction, formation, and in consolidating understanding (especially in junior and middle clases) by revealing the subject matter of the concept taught by the solution of the problem. In this way not only is the definition mastered, but acquaintance is made with the properties which follow from the definition. So formulation of the definition is not the first stage in the introduction of one or another concept. Repetition by a system of exercises is very important - the constant inclusion in exercises of matter taught earlier, but now on a new, higher level with the addition of new knowledge. An important place is allotted to exercises for developing mathematical thinking in children. Included in the system of exercises are tasks of a problematic and research character.

New in the methodics of mathematics is an exact separation of the results of teaching, which must be provided for the teaching of each point, paragraph, topic, and course in full, and the composition on this basis of a system of exercises.

Formerly this matter was dealt with by the teacher intuitively. Now in most textbooks and methodic manuals, didactic material is selected from this particular position.

Another characteristic of the course is the formation of skills and practices in sufficiently simple exercises. Complicated and burdensome exercises are excluded from the compulsory sections. The necessity of following this principle came about chiefly because of the strengthening of the role of mathematics in compulsory universal education.

Generalised analysis of material in the contemporary school mathematics textbooks can be represented in the following structure of an ideal textbook (from the point of view of organising teaching). At the beginning of each chapter the aims of studying the material are briefly formulated, and at the end there is a short summary of the basic contents, control questions on the most important material, and a list of new terms and symbols. At each point of the textbook (material is allotted for two to three lessons, the aim of teaching is pointed out. In the text basic and supplementary material and formulations are separated either by type or colour. The most important and difficult stages of the reasoning are marked by special signs. In examples of solutions of problems and exercises place is set aside where the pupil may admit a mistake. At the end of that section control questions are given on the basis of the subject matter and exercises on the compulsory part of the section. Here there are additional questions of a general character for further development; these problems are of greater difficulty and are not obligatory for all.

At the end of the book there is a list of axioms. definitions, theorems, rules, notations, indices, and sometimes a dictionary of terms used in the course.

A methodic manual for use with the textbook contains commentaries on the whole course giving the aims of its teaching. To each chapter of the textbook a corresponding commentary is given in which the fundamental methodic ideas in the course are explained, and a description of the basic organisation of the work of the pupils is given, together with the method of instruction.

Finally, the subject matter of each section is explained in great detail, with its place in the general system of formulating knowledge, skills and practices in pupils. It is desirable that here the teacher should find exercises which he can use (other than those given in the textbook) when explaining new material, when strengthening and controlling knowledge, and also showing typical mistakes which pupils might make while studying, showing their reason and giving recommendations for their avoidance or correction.

In this way the management of learning activity of pupils and teachers achieves the exact working out of the hierarchy of ideas of teaching, bringing them to the knowledge of teacher and pupils by isolating the basic results of instruction.

Professor Monakhov emphasised the points underlined in his paper, and went on to select the most important improvements in the new texts introduced in the 1970s:

1. the establishing of connections between the concepts of 6th-8th grade algebra. Methodologists are now seeking the optimum number of concepts around which to build a course.
2. the formation of basic concepts over a long period of time leading to sound knowledge; this implies a spiral approach to the construction of the curriculum. However, a new concept must be used straight away. It is no good introducing a concept in the expectation of applying it at a later stage of the spiral, as it will simply be forgotten.
3. the division of a topic into units, each supplied with a set of exercises to check progress. Much research has gone into the production of appropriate and useful exercises.
4. more support for the teacher in the teachers' books. The precise instructions as to the time to be spent on a topic are to prevent teachers over-teaching a topic they like at the expense of others.

In the ensuing discussion it was established that the USSR makes much more use of oral tests in internal examinations than is common in the UK. However, the country has no experience of the type of practical texting used by the Assessment of Performance Unit.

2b FORMATIVE PERIODS IN ENGLISH MATHEMATICAL EDUCATION : DEVELOPMENTS IN THE POST-WAR YEARS

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Up to the end of World War II the provision of public education in England (see note 1) took the form illustrated in figure 1. There was a basic education for all children between the ages of 5 and 14; at the age of 11 years, children of good academic ability had the opportunity to compete by examination for places at a grammar school. Those who were successful followed a five-year course from age 11 to 16, leading to a first public examination in a number of subjects (at least five, and rarely more than eight). At the age of 16 many children left the grammar school for employment. However, those hoping to proceed to the university or to enter certain professions could choose to remain for a further two years in the grammar school up to the age of 18, when they would take a second public examination. This last two-year stage is known as the 'sixth form'. Education in the sixth form was (and still is) more specialised, and normally only two or three subjects of the student's own choice were offered in the second public examination. Thus most children in the country had nine years of public education, whilst those of the highest academic ability might spend as many as thirteen years at school.

Several changes took place in the system of education during the next thirty years, but by 1975 it had reached the form illustrated in figure 2. By now all children are in school between the ages of 5 and 16; in some parts of the country this time is spent in two schools ('primary' and 'secondary') with a break at the age of 11, while in other areas it is spent in three schools ('first', 'middle' and 'upper'), usually with breaks at ages 8 and 12, or 9 and 13. This choice is made by locally elected councils. The first public examination at age 16 has been broadened so that it is available to children of a wider range of ability (see note 2). The most able children may take this in eight or even more subjects, whilst children of average ability may offer the examination in only a small number of subjects and at a lower academic level. There is still the possibility for those of good academic ability to continue their studies up to the second public examination at age 18 in the sixth form, though it is increasingly common for this sixth form to be in a separate college rather than part of the secondary school. The proportion of students continuing their education full-time to age 18 varies from one part of the country to another, but is typically of the order of 20%.

Against this background I propose to describe the most important changes which have taken place in the teaching of mathematics up to the level of the first public examination during the post-war period. (Changes in sixth-form mathematics will be dealt with in a later session.) However in doing this it is important to make two points clear. First, in England decisions about the curriculum are the responsibility of local education committees and are not prescribed by the central government. In practice these decisions are often delegated to the teachers in the separate schools, especially at secondary level. Uniformity of practice, inasmuch as it exists, is achieved through the system of examinations and through the popularity of certain textbooks; and, even in these respects, those schools which wish to set special examinations or to use their own working materials are free to do so. Secondly, changes which take place, either within the overall system of education or within the provision for instruction in a given subject, are normally achieved by gradual stages rather than at a fixed time. What I am to describe to you must therefore be understood in general terms as an average, rather than as applying strictly to the situation within the country as a whole (or even within any one particular school).

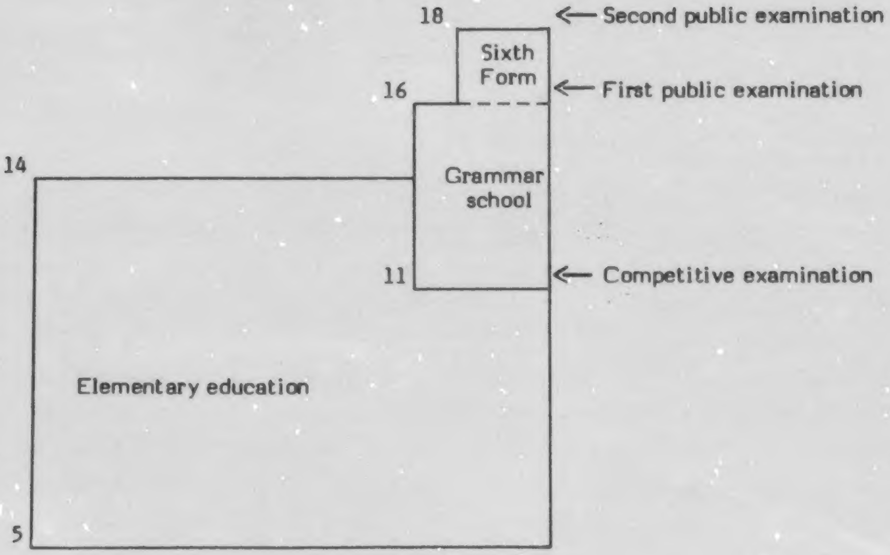

18
Second public examination
Sixth Form
16
First public examination
14
Grammar school
11
Competitive examination
Elementary education
5

Fig.1 The school system in England, 1945

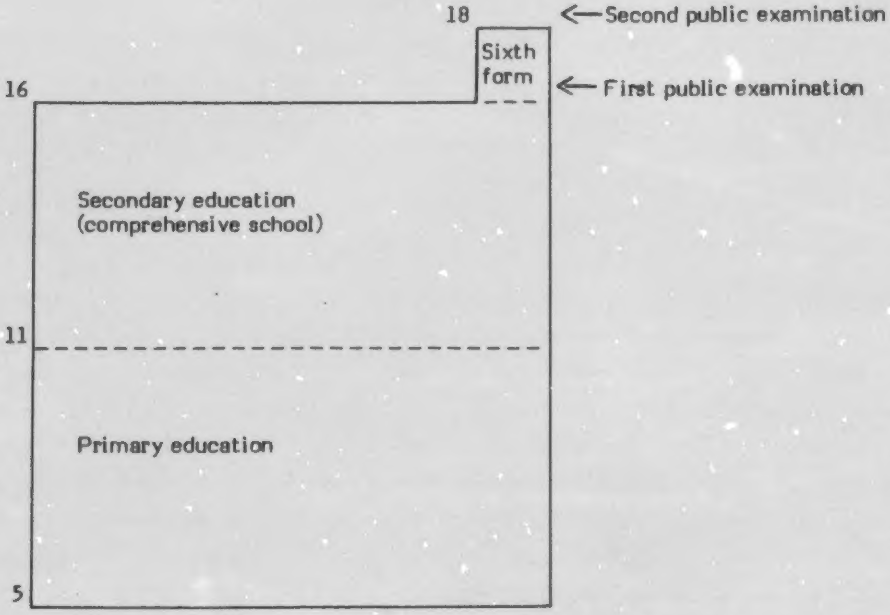

18
Second public examination
Sixth form
16
First public examination
Secondary education (comprehensive school)
11
Primary education
5

Fig.2 The school system in England, 1975

The mathematics curriculum in 1945

The basic curriculum for those children following the minimum nine-year education consisted almost entirely of arithmetic, and this was dominated by the complicated system of units of money, length, mass, etc, in use in the country at that time. Besides learning to carry out the basic arithmetic procedures on whole numbers, fractions and decimals (but not negative numbers), children learned to apply their work to typical commercial situations such as might arise in a shop or an office. Older children also did a small amount of practical geometry and learned to do calculations on areas and volumes.

In the grammar school the main course for children aged 11-16 was made up of separate components reflecting different traditions in British education:

1. The commercial arithmetic already begun before age 11 was extended to calculations such as interest on money invested, profit and loss, exchange of money into foreign currencies, etc, which would be useful to those children who left school at age 16 and took employment in business houses. This may be regarded as a legacy of the position built up by Britain during the 19th century as a major centre for international trade.

2. An academic course in geometry, developed in the tradition of Euclid's 'Elements'. Pupils would be expected to be familiar with, and to be able to reproduce proofs of, about fifty of the Euclidean theorems, and to be able to apply these by logical deduction to other simple geometrical configurations. This was a self-contained course which reflected the academic outlook of the universities in the 18th and 19th centuries, with their high respect for the traditions of classical Greece.

3. A course in algebra which was also essentially academic in its origins, though it included components of graphical work and use of formulae which owed their place largely to pressure from teachers of science and engineering. However, the main emphasis of this course was on the formal processes of the algebra of real numbers, including the solution of equations, and the application of the standard arithmetic procedures to polynomials and rational expressions.

4. A course for the older pupils in numerical trigonometry, including calculation using logarithms, which was a more recent addition to the scene and which illustrated the application of mathematics to problems of surveying, measurement and geography. This course could be regarded as an appendage to the work on arithmetic, and helped to link that with the course in geometry.

Not only did these courses represent separate traditions in British education, they were usually taught in parallel but as separate units, and the first three were usually tested in separate examinations. In grammar schools all four components were included in the curriculum of almost all boys and of a substantial proportion of girls, but the mathematics curriculum of some girls was at that time restricted to arithmetic only.

It will be helpful to discuss the changes which have taken place since that time under five principal headings: the unification of mathematics, changes in the primary school, mathematics for all children, the introduction of 'modern mathematics', and new philosophies of mathematics teaching.

The unification of mathematics

Towards the end of World War II a conference (see note 3) recommended that mathematics in the grammar schools should be taught (and examined) as a single subject combining arithmetic, geometry, algebra and trigonometry. This was important for two reasons.

In the first instance, it forced teachers, textbook writers and those responsible for examinations to think seriously about the relations between the various components of mathematics which (as explained earlier) had come into the curriculum for quite different reasons. It no longer made sense to regard geometry in isolation as a structure of interrelated theorems; consequently the learning of proofs was greatly reduced, and geometrical configurations were used as a basis for problems in arithmetic and algebra. In arithmetic less emphasis came to be placed on financial and commercial applications, and graphs were used increasingly to develop links between arithmetic and algebra. The role of algebra as a support for the teaching of science received more emphasis, and interest shifted towards formulae and the idea of functional relationship; this was taken as far as an informal introduction to the ideas of differentiation and integration, with applications to topics such as kinematics, optimisation and the calculation of areas and volumes.

The second respect in which this recommendation was important was that it provided a framework within which far more substantial changes in content were to be achieved at a later stage. It is inconceivable that the move towards 'modern mathematics' which took place after 1960 could have been effective if the concept of a unified mathematics course had not already taken root in the thinking of British teachers.

Changes in the primary school

The next area in which major changes were to be initiated was in the teaching of younger children up to the age of 11. A number of separate threads made their contribution to this development:

1. A new interest in the application of psychological and pedagogical research to the teaching of mathematics. Z P Dienes and C Gattegno both worked in this country for a period, and stressed the importance of experience with physical materials in building up knowledge of mathematical structures. (Gattegno was one of the founders of the Association for Teaching Aids in Mathematics, which in 1962 evolved into the Association of Teachers of Mathematics and which provided a focus for experiment within the community of mathematics teachers and teacher trainers.) More influential in the long term was the developmental theory of J Piaget, with its emphasis on the need for conceptual understanding in children's learning and on the role of the child's own activity in the formation of this understanding. Piaget's approach appealed to teachers reared in the English empirical tradition, and several distinguished interpreters of his work emerged in this country.

2. Improvements in the quality, quantity and status of teacher training. Some able mathematical educators were attracted to work in this field, and the lengthening of the course from two to three years in the early 1960s (with a stronger component of academic study) gave an opportunity to put mathematical work on a sounder basis. It was primarily through the teacher training colleges that the influences referred to in paragraph 1 were disseminated amongst classroom teachers.

These improvements were matched by a growing awareness of the need for the continued training of teachers in-service, and a considerable boost was given to this when teachers' centres - some of them specially designated for mathematics - were set up throughout the country.

3. The national trend towards the organisation of education in comprehensive schools, which removed the need for a competitive examination to be held at age 11 to select children for academic education, and which thus freed primary schools to take more responsibility for the development of their own curriculum.

4. A decline in the importance of calculations involving length, mass etc in mixed units, which was eventually reinforced by government decisions to adopt a decimal system of coinage and (by gradual stages) metric units of measurement. This made space available in the mathematics curriculum for the introduction of new ideas.

Mention should also be made of the influence at this time of a number of strong leading personalities in the field of mathematical education who concerned themselves with young children. The publication in 1957 of the Mathematical Association's report 'The teaching of mathematics in primary schools', and the establishment in 1964 of the generously funded Nuffield Primary Mathematics Project were important landmarks in English mathematical education.

The result of all these influences was a strong movement away from the rote learning of arithmetic processes towards a mathematical education more informal in style and far broader in content (emphasising pattern in number and shape, and practical work in measurement and estimation, leading to graphical representation), in which the pace of learning was governed by the development of the individual child rather than by an externally imposed scheme of work. Emphasis was placed on children's ability to describe their mathematical experiences verbally and on the development of an effective mathematical vocabulary. It has been estimated that at age 11 children nowadays are familiar with about three or four times as many 'mathematical words' as were the corresponding children thirty years previously.

Mathematics for all children

It has already been pointed out that, under the old system of school organisation (see figure 1), children beyond the age of 11 received a very different kind of mathematical education if they were selected for the grammar school from that which they would have received otherwise. However, with the gradual development of comprehensive schools for all children between the ages of 11 and 16, teachers began increasingly to question the rightness of having separate mathematics curricula for different groups of children. They found by experience that some of the work in geometry and algebra, which had hitherto been restricted to grammar school children, could also be taught successfully and could enrich the mathematical programme of children of a wider range of ability. This development was encouraged by the widening of the scope of the first public examination (see note 4) and by the recognition within the public at large that modern education needed to prepare children to enter a technological world whose roots would be embedded firmly in a mathematical subsoil.

One result of the move towards comprehensive education was an increase in the number of mixed-sex schools. Moreover, there was a marked change in public attitudes towards career opportunities for girls during this period. There is now no significant difference between the mathematics curricula of boys and girls in English schools.

However, within the framework of a common unified mathematics curriculum, most teachers recognise that children differ from each other in their aptitude for mathematics and - more particularly - in the pace at which their mathematical education can usefully proceed. Although some schools choose for the first year or two of secondary education to provide for children of different ability within the same class, from the age of 12 or 13 onwards the great majority of children continue to be taught in separate classes according to their mathematical ability and maturity, so that some children in the secondary school carry their mathematics much further and in greater depth than others; there has been no support for the concept of an age-related mathematics curriculum. It is also increasingly recognised that children differ in their motivation to learn mathematics; a child with little aptitude for the subject, who sees little relevance for algebra in his future career, is unlikely to perform successfully in a broadly based mathematics programme. For these reasons in the last few years there has been a tendency to restrict the mathematics curriculum of those children who have little success in the subject to those topics in mathematics whose importance is most obvious to the pupils.

The introduction of 'modern mathematics'

The last two sections have described aspects of the mathematics curriculum which have been the subject of slow change over a long period of time. The changes now to be described represent a much more sudden discontinuity in the development of the mathematics curriculum, and in their most active phase are associated particularly with the years 1960 to 1970. However, it must again be emphasised that any decision to adopt the new curriculum was made separately by each school, so that its uptake on a national scale took place more gradually, and indeed there are still many schools which have not chosen to implement it, or which have adopted a compromise in which a few of the new ideas have been grafted on to the old curriculum.

In certain respects what happened in Britain was a part of a wider international movement and reflects a growing desire amongst mathematics educators to share ideas with colleagues from other countries. For example, the introduction of some of the language of sets (and even some of the notation of set theory) in both primary and secondary school teaching, as an aid to the presentation of certain mathematical concepts, was a development to which parallels could be found in Europe and the United States of America. Other examples include the presentation of the concept of a function as a mapping (with an associated arrow notation), the attempt to introduce greater precision of mathematical language (for example in distinguishing the operation of subtraction from the designation of a negative number), and the increased emphasis on structure in algebra. However, these changes received less than wholehearted support from many British teachers, who were somewhat suspicious that they represented an undue dominance by pure mathematicians. They were also unpopular with teachers of science who wished to make use of mathematics in their own classes.

There were two other influences in England which strongly affected this new wave of change in the mathematics curriculum. The first was an awareness of new ways in which mathematics was being applied in industry and commerce, especially under the influence of the electronic computer. The teachers who prepared the new textbooks were conscious of the growing importance of statistics, of operational research techniques such as linear programming, of the application of Boolean algebra in electrical engineering and computer design, and of the increasing use of numerical methods and flow-charting techniques. All of these they sought to introduce into the new curricula, largely at the expense of outdated arithmetical applications and the more technical aspects of algebra.

There was also considerable dissatisfaction with the unified mathematics curriculum in existence at the time, which (as described above) was a somewhat unsatisfactory blend of material drawn from widely different traditions. New publications which had recently appeared at a more advanced level suggested ways in which a more genuinely unified programme of geometry and algebra could be effected. The keystone of this was the replacement of the conventional static Euclidean approach to geometry by an interpretation in terms of transformations and vectors. For younger children this offered the possibility of a programme of active experimental work; leading on from this, the introduction of co-ordinate representation and matrix notation provided a basis for studying the combination of transformations by geometric and algebraic methods in parallel. Such a programme also opened up possibilities for links with the arithmetic of proportion and ideas in trigonometry, and could be shown to provide illustrations for a variety of applications of mathematics. These ideas were successfully implemented in many classes, although they proved unduly sophisticated and lacked appeal for pupils of lower ability. Moreover, those teachers who did not have a strong mathematical background tended to teach this new material according to the letter of the book, with little appreciation of the reasons for the change.

Finally, note should be taken of the new range of apparatus for teaching mathematics which came into schools. For arithmetic calculation, the use of logarithm tables had long been accepted in the mathematics classroom; now mechanical calculating machines and simple slide rules were introduced. (All of these are now being supplemented by electronic calculators.) For geometrical work geoboards, tracing paper, parallel strips, paper with a variety of rulings, and scissors were added to the already familiar set square, compass and protractor.

This is indicative of the new spirit which began to pervade mathematical education at this time.

The result of these combined influences has therefore been the production of a distinctively British modern mathematics curriculum which, though not universally accepted in all schools, has secured a sufficiently wide appeal amongst mathematics teachers to suggest that it will form the basis for future developments in this country.

New philosophies of mathematics teaching

This survey would not be complete without mention of one further important change, which does not relate to the content of the mathematics curriculum but to the way in which teachers view mathematics as a subject within the curriculum. At the beginning of the period covered in this paper, one could say that there was a general consensus amongst mathematics teachers as to the purpose of the work in which they were engaged. Whilst there might be differences of opinion about the detail of the curriculum, teachers saw their role as being that of instructors with certain knowledge and technical skill which they had to pass on to their pupils. For the more able pupils this would include the technique of devising geometrical proofs and training in strategies of algebraic computation; but the material was given, and the pupils were expected to learn it.

Many British teachers nowadays see the aim of mathematical education in very different terms. Whilst there remains some material to be learned, mathematics is treated more as a experimental science and less as a set of deductive procedures. Children are encouraged to explore potentially mathematical situations freely to observe patterns, and to make conjectures on the basis of experiment. They are encouraged to make generalisations from their tentative conclusions. Absolute standards of proof have been replaced by a variety of levels of proof-argument through which the child will develop as he grows to mathematical maturity. In short, 'learning mathematics' has come to involve (and not just for children of the highest ability) learning to behave as mathematicians behave.

For some teachers a corollary of this changed attitude has been to abandon the textbook as a medium of mathematical education, and to replace it by a set of cards each covering one particular item of study. Textbooks are thought to imply for the child that all answers are known in advance, but it is part of this new philosophy of mathematics teaching that the student should have some say in the direction to be taken by his mathematical enquiries. This is still a minority viewpoint amongst teachers of mathematics in England, but it is undoubtedly exerting an important influence on our thinking about the mathematics curriculum of the future.

Conclusion

This paper has sought to identify the major influences for change on the British mathematics curriculum in the postwar period. It is essentially a personal interpretation, since it is in the nature of British education that curriculum change occurs not through a sequence of historic events but through a continuing exchange of ideas amongst leading exponents of mathematical education, both within and outside the school. This paper is therefore offered not as an authoritative account but as a basis for discussion and clarification.

Notes

1. Arrangements for education in other countries of the United Kingdom differ in certain respects from those in England. The system in Wales is very similar to that in England, but education in Scotland has evolved from different traditions. Although many of the developments described in this paper had their parallels in Wales and Scotland, the account given here is based entirely on experience in England.

2. This broadening was effected by adjoining to the original examination for the General Certificate of Education (Ordinary level), which was designed for pupils in grammar schools, a second examination known as the Certificate of Secondary Education, in which a lower standard of achievement was acceptable. However, at the present time steps are being taken to combine these two examinations into a single system, covering the needs of pupils over a wide range of ability.

Furthermore, the rules were changed to allow pupils whose performance only reached the required standard in one or two subjects to take the examination in these subjects alone.

3. This 'Conference of Representatives of Examining Bodies and Teachers' Associations' met under the chairmanship of Professor J B Jeffrey, and its report is usually referred to as the 'Jeffrey Report'. It should be noted that there was no representation of national or local government at the conference, and that its recommendations were implemented on the initiative of the examining bodies alone.

4. When the Certificate of Secondary Education (see note 2) was set up, it was envisaged that 'mathematics' would be examined as a unified subject, not simply as arithmetic. This remains the general practice, although some examinations in arithmetic only (taken mostly by the pupils of lower ability) have subsequently been established, largely as a result of pressure from employers of 16-year-old school leavers.

Mr Quadling drew attention to the way in which changes are made in British education, using the changes in mathematics teaching described in his paper as examples.

The changes in the primary school came about when an interest in the psychology of learning and in learning by practical activities coincided with a period of great freedom for schools to plan a new curriculum. This occurred when the central government policy of comprehensivisation of secondary schools resulted in the abolition of the 11+ selection examination, the content of which had been determined by each LEA and had in turn largely determined the mathematics curriculum in schools.

The move toward unification of the syllabus began in some schools and spread. It was helped on in some cases by the advice and in-service training provision of the LEA, but the decisions of central government played little part in it.

In discussion of the section of the paper 'Mathematics for all children', it became apparent that, while the provision of an appropriate mathematical education for a wide variety of pupils is a major preoccupation in Britain, the Soviet participants do not talk about 'different ability-levels'. Their apparent success at education practically every child effectively, using a common syllabus and a common speed of progress through it, became one of the major points of interest to the British participants during the seminar.

3a INQUIRING INTO THE TEACHING OF MATHEMATICS IN SCHOOLS

Dr W H Cockcroft, Vice-Chancellor, The New University of Ulster

Introduction

In July 1977 the Education, Arts and Home Office Sub-Committee of the Parliamentary Expenditure Committee published a report in which it was stated 'it is clear from the points which were made over and over again by witnesses that there is a large number of questions about the mathematical attainments of children which needs much more careful analysis than we have been able to give during our enquiry. These concern the apparent lack of basic computational skills in many children, the increasing mathematical demands made on adults, the lack of qualified maths teachers, the multiplicity of syllabuses for old, new and mixed maths, the lack of communication between further and higher education, employers and schools about each group's needs and viewpoints, the inadequacy of information on job content or test results over a period of time, and the responsibility of teachers of mathematics and other subjects to equip children with the skills of numeracy'.

It made as 'possibly the most important of our recommendations' that the Secretary of State for Education and Science should set up an Inquiry into the teaching of mathematics.

In their reply presented to Parliament in March 1978, the Government agreed 'that issues of the kind listed in the Committee's report need thorough examination' and announced their decision to 'establish an Inquiry to consider the teaching of mathematics in primary and secondary schools in England and Wales, with particular regard to its effectiveness and intelligibility and to the match between the mathematical curriculum and the skills required in further education, employment and adult life generally'. They further undertook that the Inquiry would examine the suggestion that there should be a full analysis of the mathematical skills required in employment and the problem of the proliferation of mathematics syllabuses at A level and at 16+.

The Committee was set up in August 1978. There were eighteen members who came from all levels of education and included representatives from industry. The Committee's terms of reference were 'to consider the teaching of mathematics in primary and secondary schools in England and Wales, with particular regard to the mathematics required in further and higher education, employment and adult life generally, and to make recommendations'.

The work of the Committee

The Committee met for the first time in September 1978 and continued to meet at three-weekly intervals thereafter. It concluded its work and presented its report to government in 1981.

At an early stage the Committee decided to obtain evidence and informed opinion from as wide a variety of sources as possible. Letters were sent, inviting evidence, to some 550 associations, colleges, local authorities, employers and other bodies.

The Committee was greatly encouraged by the helpful response to its requests for evidence and by the welcome that many people gave to the setting up of the Inquiry. Some 850 pieces of written evidence were received including a significant number from bodies and individuals who were not approached directly; many were both extensive and carefully argued. In many cases submissions had been prepared by groups especially set up for the purpose within a variety of bodies and institutions. A number of submissions included papers and reports that had been prepared before the Committee was set up and showed that much consideration had already been given to the mathematical needs of pupils both for employment and for adult life. Evidence from companies often included examples of entrance tests that were used.

Research projects

When considering how best to implement its terms of reference the Committee became aware of a need to have access to more detailed information in certain areas than was likely to be received in written evidence. It therefore requested the Department to commission four research projects.

1. The mathematics required in employment

This project was based at the University of Bath under the director of Professor D E Bailey, assisted by Mr A Fitzgerald of the University of Birmingham who, at the request of the Committee, had been seconded to Bath for the duration of the project. It concentrated mainly, though not exclusively, on the mathematics needed by 16+ and 17+ school-leavers. After discussion with interested and informed bodies and individuals and after studying the relevant evidence submitted to the Committee the research workers studied first-hand the needs of industry and commerce. The firms visited included large and medium-sized companies and also included a random sample of small firms. One of the outputs of the work was a series of case studies of applications of school mathematics in various forms of employment.

2. Types and levels of mathematical needs in employment

Based at the Shell Centre for Mathematical Education, University of Nottingham, under the direction of Mr R L Lindsay, this project complemented the Bath project by undertaking in the first instance an in-depth study of the mathematics required at clerical level within the Boots Company of Nottingham and at a later stage within a number of other companies. It also accelerated studies which had already been in progress at the Shell Centre in the fields of engineering, agriculture and the Health Service.

3. The mathematical needs of adults in daily life

This was undertaken on behalf both of the Committee and of the Advisory Council for Adult and Continuing Education. It endeavoured through interview and observation to establish a typology of the most basic mathematical problems regularly encountered by adults and the variety of strategies and methods used to cope with these problems. As a result of the work a Gallup Poll survey was undertaken covering the whole of England and Wales.

4. A critical review of existing research on the teaching and learning of mathematics

This was undertaken by Dr A Bell of the Shell Centre for Mathematical Education and Dr A Bishop of the University of Cambridge. It collated and presented in a form suitable for future publication and for use by the Committee, existing theory and research evidence relating to the teaching and learning of mathematics.

The Steering Groups for all of these projects included members of the Committee and relevant evidence which had been received by the Committee was made available on a confidential basis to those engaged on the projects.

Visits to schools

Members of the Committee were in no doubt that they needed to visit schools of all kinds. The Committee therefore decided to undertake a series of visits to schools chosen on the basis of size, type and geographical situation, eg inner-city, small industrial town, rural area, etc. The purpose of the visits, usually made by two members, was to observe and to listen and members spent as much time as possible in classrooms. They were also able to invite comments relating to the terms of reference of the Committee from teachers in these schools and it was clear that many teachers welcomed this opportunity. In all cases, schools were most helpful; in some instances the local mathematics adviser took the opportunity to join in discussion with members at the end of the day. Thirteen primary, two middle and fourteen secondary schools were visited. A small number of visits were also made to schools where work of particular interest to the Committee was going on.

Visits to industry

Visits were made to a number of companies chosen to represent a range of different types of employment. During these visits companies were asked in particular to provide an opportunity for the members who were visiting to talk to young employees who had recently joined the company either directly from school or after some kind of further training as well as to supervisors, foremen and those responsible for training. The visits were co-ordinated with those which formed a major part of the work of the two research projects investigating the mathematical needs of employment.

Working Groups

Six Working Groups were set up, each consisting of approximately six members and each concerned to consider in detail a major area of the work of the Committee. The Groups gave detailed consideration to matters relating to their areas of concern; they studied the relevant written evidence, prepared discussion papers for the full Committee and made suggestions as to bodies and individuals who should be invited to give oral evidence.

The over-lapping membership of the Groups enabled liaison to be maintained especially in areas of concern to more than one Group, eg continuity between the various phases of mathematics, induction and in-service support for teachers and the work of the middle schools. What follows gives an indication of matters to which each Group gave attention.

1. Working Group on Primary Mathematics

Areas of study by this Group included the primary curriculum and guidelines for teachers issued within local authorities, method and organisation of mathematics teaching, the relationship between understanding and skill, the place of mental calculation, the influence and use of calculators and continuity between phases of education. In addition to its discussion of relevant evidence the Group studied the report of the National Primary Survey, the Schools Council publication Mathematics and the ten-year-old', and the document 'Mathematics 5-11' published in the HMI 'Matters for discussion' series. They found most useful the reports published by the Assessment of Performance Unit, concerned with the performance in mathematics, over the whole of England and Wales, of 11-year-old children.

2. Working Group on Secondary Mathematics

This Group studies all aspects of the secondary mathematics curriculum as a whole both for those who leave school at 16 and those who leave at a later age. It considered the variety of methods of organisation and staffing, the time allocation for mathematics teaching, the role of the head of department and the evidence relating to the possible introduction of a 'core curriculum' in mathematics. It also considered the relationship between the mathematics studies in schools and that required for future education and employment. Once again they found most useful the survey of secondary schools, which had been published while the Committee sat, and again the report of the APU concerned with the mathematical abilities of 15- and 16-year-old children.

3. Working Group on Examinations

This Group studied the pattern of public examinations then currently in use, various modes of examining, the variety of syllabuses that existed and the suggestions that had been made to the Committee relating to the provision of a core syllabus at both 16+ and 18+. It held meetings with representatives of some CSE Boards to discuss patterns and modes of examining and sought information from GCE Boards about such matters as the possible reduction of the number of syllabuses then on offer, the use of calculators and formula sheets and the provision of single and double subject examinations at A level. It also sought to obtain statistics from the Boards and from the Department about examination entry and performance and to identify changes in the use of examinations by schools in the immediately preceding years.

While the Committee was sitting there were published proposals for a common system of examining at 16+ and this naturally led to discussion by the Committee of the ways in which mathematics should fit into such a system.

Almost inevitably the Working Group on Secondary Mathematics and the Working Group on Examinations found themselves having joint meetings during the last year or so of the period of the Inquiry.

4. Working Group on In-Service Training and other Support Services

Areas of study by this Group included the needs of teachers of mathematics for in-service training, the variety of provision that existed and the extent to which it met the current needs. It sought to identify good practice in this field and was concerned to establish whether, and if so by what means, the effectiveness of in-service training could be assessed and the financial implications of effective provision. It also investigated other methods by which support for the teacher could be provided both inside and outside the classroom.

5. Working Group on Initial Training

The Group considered the extent to which the need for teachers of mathematics, in terms of both quantity and quality, had been satisfied in the immediately preceding years and how it could be satisfied in the future. In particular it considered the nature of the initial training provided both for mathematics specialists and for those, especially intending primary teachers, who, although not specialists, would nevertheless be required to teach mathematics. It also considered the arrangements which were made for induction into the profession.

6. Working Group on Mathematical Needs in Employment

The Group considered the very wide range of evidence that had been received from employers, professional associations and industry training boards. It maintained close contact with the research projects on the mathematical needs of employment undertaken at Bath and Nottingham and representatives of these projects attended some meetings of the Group.

Statistical information

It became clearly necessary for the Committee to obtain statistical information about a variety of matters, in particular the size and deployment of the teaching force then in schools, current and future recruitment needs, numbers studying mathematics at universities and colleges and a number of matters relating to examination entry and performance.

Among the existing sources were:

a. information collected each year by the Department from schools and colleges;
b. information collected by the DES Sample Survey of Secondary School Staffing in the autumn of 1977;
c. information collected during the National Primary and Secondary Surveys;
d. information which could be made available by examination boards;
e. information which was available from the Universities' Statistical Record.

A limited amount of statistical information also came to the Committee in evidence.

Each of these sources, in its own way, presented the Committee with problems. Thus, for example, the information which was collected each year from schools had not hitherto been analysed in ways which were subject-specific and the computer programs which were needed to do this did not exist. Again, the Committee sought, from various surveys that had been undertaken, information which required the raw data to be analysed in ways which had not been forseen and for which, because the surveys were planned before the Committee was set up, no provision had been made. A less important, though not insignificant, problem was that both computer studies and statistics had traditionally been included as part of mathematics for the purposes of statistical analysis and it was not in general possible to obtain separate figures relating to these two fields.

Mathematics teaching outside England and Wales

Although it was not part of the task of the Committee to make a comparative study of mathematical education throughout the world, it was thought likely that some knowledge of practice in countries which were socially and economically comparable with the United Kingdom might inform its discussions. The Committee therefore sought information about mathematical education in a number of other countries. It was able to obtain statements of official policy but it proved more difficult to obtain information about practice in the classroom.

However, with the help of the Central Bureau for Educational Visits and Exchanges and the League for the Exchange of Commonwealth Teachers it was possible to approach a number of British teachers who had taught overseas and also some teachers from other countries who were then teaching in England and Wales and the Committee received some very helpful information. It also received help from the ritish Council and from people in other countries who were known members of the Committee. In addition, information and assistance was received from the Scottish Education Department.

Editor's note

At the time of the seminar, the Committee of Inquiry had not yet reported its findings to the Minister at the Department of Education and Science. In the event, Dr Cockcroft was unable to be present at the seminar personally. In his place Mr John Hersee, a member of the Committee, amplified Dr Cockcroft's paper, concentrating on three aspects of mathematical education in England and Wales on which attention is currently being focused. A summary of his talk follows.

3b MATTERS OF CURRENT CONCERN

J Hersee, member of the Committee of Inquiry

Speaking instead of Dr Cockcroft, I can offer only a personal view of what seems to me the major current concerns in mathematics education in England. Nothing that I say should be interpreted as a hint of the findings of the Committee of Inquiry, with which Dr Cockcroft's paper deals.

It might be argued that, because our 'system' of education is so unstructured, we have reached a state in mathematics where we need a Committee of Inquiry to sort out our problems! Be that as it may, I shall consider three main headings.

Teachers We have a considerable shortage of teachers who are adequately qualified in mathematics, both in primary schools (where the teaching is by the 'class teacher') and in secondary schools (where the teaching is by a specialist). For all teachers, the period of initial training is too short to equip a teacher for a lifetime in the mathematics classroom. A teacher's certificate does not specify the subjects he may teach, nor the level to which he may teach them.

We need a proper and regular pattern of in-service training for all teachers of mathematics:

- to assist teachers whose mathematical background needs strengthening
- to help all teachers of mathematics to develop better teaching methods and approaches throughout their careers.

What shall we teach? The content of the school curriculum is academically biassed, as we have heard from Dr Howson. Yet there is no doubt that one main reason for devoting so much of the school timetable to mathematics is because of its usefulness. So we expect children to leave school equipped with some mathematical knowledge and skills. Complaints from employers that young people often lacked this knowledge and these skills were a major factor in the setting up of the Committee of Inquiry.

We now have documents which are relevant to these issues:

1. The work of A Fitzgerald and R L Lindsay gives detailed information about the actual mathematical demands of many jobs.
2. The Concepts in Secondary Mathematics and Science (CSMS) report (published under the title 'Children's understanding of mathematics'), and the Assessment of Performance Unit (APU) results, show us what children are achieving now and, by implication, suggest the degree of success of present syllabuses and approaches.

The challenge is to 'match' the curriculum to the pupil, and this, I believe, means a 'differentiated curriculum', especially in the secondary school: that is, a curriculum where pupils do not all study the same content, nor is the teaching approach the same for all. In particular, we do not know what is a suitable curriculum for low attainers in mathematics.

Teachers need in-service training to develop suitable curricula.

How should we teach? It is easy to observe the changes in syllabus content that have taken place in many schools over the last 20-30 years. There is less evidence, although there is some, of changes in teaching methods and approaches.

How do we change teaching methods and approaches? We need agreed aims and, at present, we either ignore aims or do not formulate them with sufficient care.

Again, we see a need for in-service support for the teacher.

Our loosely structured system throws much responsibility on the teacher, yet we seem to provide him with less support than our Soviet colleagues give to their teachers.

The discussion which followed was on the needs of low attainers, and the relation between aims and methods.

Low attainers in the UK are considered to be the 30% of pupils who do not attempt either CSE or GCE O level examinations. Differences in ability are emphasised more in the UK than in the USSR, but a distinction is now being made between the rate at which a child learns and his capacity for learning. Traditionally in the UK, syllabuses have been planned for the most able and then reduced in content for the less able. In the USSR the syllabus is planned for the less able and supplemented for the more able. Both countries have need of a course usable by the whole ability range. In the UK some teachers think this is possible up to age 13, while in the USSR it is done up to age 15.

Some UK teachers think that children should learn maths by a process of exploration. In the USSR, in spite of prescribed courses, such heuristic methods are encouraged by the Academy but are not proved to give 'better' results, and in some cases, have shown 'worse' results for problem solving. This led back to the conclusion that it is necessary to determing aims first and then relate teaching methods to them.

3c BASIC PUBLICATIONS ON MATHEMATICS EDUCATION FOR THE USE OF THE TEACHER

N A Yermolaeva, Deputy Director, Department of Schools, Ministry of Education of the USSR

Among issues in the improvement of education at the present time, the creation of teaching materials in all subjects in the school curriculum, including mathematics, is especially important.

In such teaching materials are included a curriculum, a textbook, didactic materials, reading books for pupils, academic and methodic literature for the teacher, cinematic equipment, slides, tables and other teaching and methodic facilities, for use by pupils and teachers in the process of teaching one or another school subject.

The curriculum is the basic government document which defines the subject matter of mathematical education.

It is formulated by academic research institutes of the Academy of Pedagogic Sciences of the USSR with the co-operation of the academic staff of Academies of Science of the USSR, with methodic-practitioners and teachers, and is confirmed by the USSR Ministry of Education.

The curriculum contains a collection of themes to be taught in secondary schools, listing the demands for knowledge and skill of pupils in each class, recommendations for the assessment of the knowledge of the pupils, and a list of teaching equipment and literature to help the teacher. The list of themes contained in the curriculum is divided into topics, into classes, with an indication of the time to be given to their teaching in conformity with the teaching programme.

The curriculum is the basis for the creation of a set of textbooks by subject. The textbook is the book for pupils of the course, a system of problems and questions, which give necessary practice. At the present time in the secondary school the following mathematical textbooks are in use: 'Mathematics I, II, III' by a group of authors - M I Moreau, M A Bantova, G V Beltiukova, A S Pchelko, A M Pishkalo; 'Mathematics IV, V' by I Y Vilenkin, S I Shvartsburd, K I Neshkov, A S Chesnokov and T F Nechaeva; 'Algebra VI, VII, VIII' by Y N Makarichev, N G Mindiuk, K S Muravin and S B Syvorova; 'Geometry VI-VIII' by A N Kolmogorov, A F Semionovich and R S Cherkasov; 'Algebra and the elements of analysis IX-X' by A N Kolmogorov, O S Ivashev-Mustatov and S I Shvartsburd; 'Geometry IX-X' by V M Klopsky, Z A Skopets and M I Yagodovsky.

Several books have been published in order to acquaint the teacher with the contemporary scientific treatment of different mathematical topics, among them those included in the school course. Among these can be noted a series of articles: A N Kolmogorov 'The scientific basis of the school mathematics course', published in the journal 'Mathematics in school', 1970-72; the book 'Basic concepts of the contemporary school mathematics course', by Y M Kalyagin, G L Lukankin and others; 'Mathematics IV-I class: theoretical course' by N Y Vilenkin and others; 'Vectors in the school geometry course' by V A Gusev, Y M Kalyagin and others; 'On the way of renewal of the school mathematics course: a collection of articles and material' composed by A I Markushevich, G G Moslova and R S Cherkasov; 'What is new in the mathematics course for secondary schools' by N A Yermolaeva and G G Moslova.

In order to improve the level of the academic-methodic and pedagogic preparation of teachers, they are helped to master the methods of work in the new curricula and school manuals which are published as fundamental methodics for the whole course of the secondary school and methodic guidance to the textbook for each class. The authors of the methodic guidance are, as a rule, the authors of the textbooks and are experienced in methodics and practical work. These manuals to no small degree further the methodic preparation of teachers. They contain recommendations for the planning of material, for the learning of each topic of the course, possible variants in presentation of theoretical information, solutions to problems and exercises, and examples of homework. A large place in the manuals is given to recommendations for the organisation of individual work with pupils who are succeeding poorly and with those who show special interest in mathematics. A system of teaching and control work is given in these manuals. Methodic guidance played a significant role in the first stage of teachers' mastery of the new curriculum.

As well as this, books are produced on general and particular methodics. In them are elucidated methodic approaches to the formation of the most important concepts of the course, recommendations are made for teaching the main themes and the organisation of work with the basic kind of exercises, showing the place of each section in the general system of teaching the subject. It gives the characteristics of the development of the through-lines of the course. It is there that the special features of the textbook are expanded (structure, methodic principles of presentation of theoretical material, principles of constructing the system of exercises). An appointed place in these books is given to the exposition of the aspect of moral education in school courses, and to the polytechnical dimension, and the connections between the algebra and geometry courses are elucidated in successive primary, 8-year schools and secondary schools.

In 1975-80 a series of books on methodics was published, for example: 'Methodics of teaching mathematics in I-III classes' by M I Moreau and A M Piskalo; 'The teaching of mathematics, IV-V classes' by K I Neshkov; 'Teaching algebra in VI-VIII classes' edited by Y N Makarichev and N G Mindiuk; 'Teaching geometry (VI-VIII classes)' by V A Gusev and others; 'Teaching algebra and the elements of analysis (IX-X classes)' edited by E G Glagoleva and others; 'Teaching of geometry (IX-X classes)' edited by Z A Skopets and others; 'Methodics of teaching mathematics in secondary school' by Y M Kalyagin and others; 'On improving methods of teaching mathematics' edited by Kramov; 'The equipment of the mathematics room' by V G Boltyansky; 'Out of class mathematics work (VI-VIII classes)' by I A Orlov and others.

One of the ways of raising the effectiveness of mathematics teaching is the intensification of the independent activity of pupils in class. With the aim of concentrating the attention of the teacher on the more important sections of the course, to help him organise independent work and to give a reference point for the demands for knowledge and skills in pupils, 'didactic material', as it is called, has been published (for mathematics, algebra, algebra and the elements of analysis, and geometry) for each class and subject. It is a collection of independent and control work in several variants. The content of the work is closely connected and agrees with the content of the text being explained and with the exercises of the standard textbook and methodic manual for teachers. As well as 'didactic material', several collections of problems and exercises have been published which, to a lesser extent, are linked with the set textbook and are used by teachers for deepening and extending the knowledge of pupils, for differentiation of teaching, and for revision of the entire subject-matter studied.

A special place among teaching-methodic material is occupied by books which elucidate the subject matter and methodics of organising extra optional studies. Before the introduction of new curricula for the basic course there were separate topics in the new curricula, which formerly were not studied in school, but which it was proposed to study in optional studies. The same teacher who took the optional study with a group of pupils could earlier have prepared them for the introduction of new educational subject matter.

After completing the transition to the new curricula, topics were offered for optional studies which enlarged or deepened the systematic course. As well as this, curricula for optional studies include special courses not connected with the basic contents of school education. The most popular of these courses are: 'Computer mathematics', 'Programming', 'Mathematical analysis'. The books which have proved most useful for pupils are 'Programming for one-address machines', by P T Pesnikovsky and V N Monakhov, and 'Mathematical analysis' by N Y Vilenkin and S I Shvartsburd. Manuals for teachers for these courses contain both theoretical information and recommendations for its teaching. Among these books for teachers it is worth noting the manuals 'Computer mathematics' by S P Pulkin, 'Programming' by V M Monakhov and 'Programming and computer mathematics' by R S Ruter, B V Ovchinsky and P T Resnikovsky.

At the present time a number of long term experiments are being conducted in Soviet schools. Specially interesting are experimental curricula and textbooks in which an effort has been made to create a single course of mathematics from the first entrance class of the secondary school (without separating algebra and geometry). In the books of the Estonian pedagogues E Etverk, O Prinits, K Velsper, K Ariva, A Telgmaa, E Nurk and others is elucidated the methodics of teaching mathematics in the IV-X classes. In a number of territories of the country an experimental verification is being conducted of some trial textbooks written by the academic mathematicians and pedagogues A V Pogorelov, L S Atanasyan, T A Alimov, Y V Sidorov, Y M Kaliagin, A D Aleksandrov and others. Acquaintance with the contents of the experimental textbooks allows the teachers better to find their bearings in the ways of the developing sciences, and to take part with the authors of the trial textbooks in the search for new methodic ways of teaching. An analysis of the contents of the experimental courses, their expert evaluation and the systematic study of the condition of the pupils make it possible to define the content of mathematical education in the 1990s.

Mrs Yermolaeva outlined the place given to mathematics teaching within the Soviet system. The subject is taught by class teachers in grades 1-3, by specialist mathematics teachers in grades 4-5, and by specialist algebra and geometry teachers in grades 6-10. The syllabus and textbooks for grades 1-3 are determined by the Union Republics in accordance with recommendations from the USSR Ministry of Education, which is responsible for syllabus and texts for grades 4-10. Vocational and general secondary schools use the same grade 9 and 10 books, and the same standard is expected of all these secondary schools. The content of the course and the time allowed are prescribed, but teachers are free to use what methods they like as long as they don't distort the course. Teachers' books, written by the same authors as the pupil texts, give recommendations for their use.

From 1964 to 1968 many changes occurred but since then the syllabus has stabilised. Four alternative 5th grade texts were tried and widely discussed before the most successful was adopted throughout the USSR. In the higher grades other ways of teaching geometry and algebra were explored, although the alternative texts were fewer. At present an experiment in integrating the whole course is being tried in Estonia. There are initial attempts to integrate the course, based on mapping and reflection, which are not yet researched or worked into a text.

Since 1978, the Academy of Sciences has criticised the textbooks in use, and about twenty new texts are being trialled in small numbers of selected schools.

NB

The recent criticisms are spelt out in detail in an article by Academician L S Pontryagin, published in 'Kommunist', no.14, 1980, p.111.

Editor's note on the Soviet use of certain key words.

Algorithm

This appears to have a wider connotation than is usual in Britain. Applied to problem-solving, it refers to a systematic way of teaching children to tackle certain kinds of problem, and includes aspects of what in Britain would be described as a problem-solving strategy, as well as incorporating the standard mathematical processes involved. It can also be applied to, eg, learning a piece of mathematics, or acquiring a concept. In each case, careful analysis is necessary to discover the steps involved in the process, these steps then forming the algorithm.

Soviet practice is to study textbooks and other materials to ensure that they conform to such algorithms.

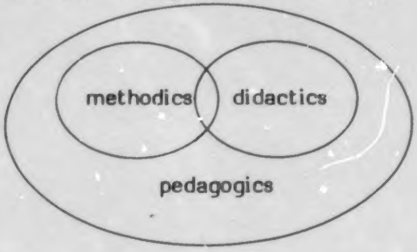
Ontodidactic

This is defined in paragraph 6 of Paper 7a.

Methodics

The principles of teaching a particular subject. It is much broader than the English use of 'methods'.

The following diagram was produced by the Soviet participants in an attempt to clarify our understanding



Pedagogics = child study in general
Didactics = general principles of teaching
Methodics = principles of teaching a particular subject. It is not a subset of didactics, as it also takes account of the nature of the particular subject.

4a THE CURRENT STATE AND FUTURE PROSPECTS OF MATHEMATICS TEACHING IN PRIMARY SCHOOLS IN THE USSR

A M Pyshkalo and V V Firsov

The reform of the entire system of mathematical education in the USSR which was carried out in the 1960s and 1970s has had a considerable effect on mathematics teaching in primary schools. The old unchanged arithmetic course, which had been put together at a time when the primary school was a self-contained phase of elementary education, had ceased to be satisfactory for schools planning for the transition to universal secondary education.

The old course set out to meet purely practical objectives, concentrating almost exclusively on training the ability to carry out the four arithmetical functions with whole, non-negative numbers, and on practising the related skills. Many written problems, occupying much space in textbooks and much time in class, provided applied counting and calculation practice. These problems were solved by arithmetical means which were often rather artificial. But these methods were at the time virtually the only means of developing mathematical thinking and acuity in children. The use of oral calculation in a large number of cases further contributed to this situation.

It should be noted at once that this did in fact establish a sound foundation for further work with numbers (and consequently also for the study of algebra), and did train certain of the pupil's personal qualities and, finally, constituted a smoothly functioning system ensuring the attainment of the designated objectives. And so the reform of mathematics teaching at the primary level, which was carried out with a view to meeting the requirements of continuity between schooling levels, had to retain what was best in all that had been achieved earlier.

The reform of the primary course in mathematics was on the following fundamental lines:

a. enrichment of the course's mathematical content and provision for continuity in the teaching of mathematics in the junior and secondary phases of schooling;
b. creation of preconditions for developing, on-going teaching, based on increasing the part played by theoretical generalisations in mathematics teaching in its initial stage;
c. substantial reduction of teaching time devoted to studying the initial course of mathematics, while maintaining demands for standards as regards the level of pupils' practical skills.

Success in finding a satisfactory solution to the problems arising in this connection flowed directly from the fundamental works produced by a whole pleiad of outstanding Soviet teachers and psychologists (L S Vysotsky, N A Menchinskaya, L V Zankov, D B Elkonin, V V Davydov et al), who demonstrated the inadequacy of traditional views on the mental capacities of junior pupils and showed that it was necessary and desirable to raise the scientific level of teaching for primary school pupils.

As a result of the reform the four-year arithmetic course (which even so had not completed the study of arithmetic, which was previously continued in the 5th and 6th classes) was replaced in primary schools by a three-year 'mathematics' course, the very title of which emphasised the change in the teaching objectives.

The basis of mathematics teaching in the reformed course is, obviously, calculation. But in the handling of traditional arithmetical questions the correlation between theory and practice has been changed, so that the formation of calculation algorithms is now placed on a definite theoretical basis. We shall quote several examples showing how theoretical knowledge and general concepts have transformed the traditional approach to the exposition of arithmetic.

An important element in the study of arithmetic is numeration. It is a traditional minimum requirement, fully providing for the achievement of purely practical goals, that pupils should be acquainted with numeration up to and including the ability to read and write natural numbers and identify the decimal component in number. Such knowledge is clearly a sufficient basis from which to move to studying the algorithms for performing the arithmetical functions. Nevertheless, to put the fundamental lines described above into effect required that the investigation of numeration problems be based not so much on dogmatic exposition as on generalisations from facts observed by the children themselves. The kind of deduced generalisations envisaged were those which would make it possible to transfer knowledge acquired in dealing with smaller numbers into the constantly expanding field of numbers being studied.

Thus even when numbers from one to ten are being studied, children are made conscious of the general rule under which every successive number in the natural series is formed. Subsequently, when studying numbers greater than 10, and later greater than 100, this rule is deliberately invoked as part of the necessary reasoning. The same applies to the formation of ideas about relationships between intermediate and allied numbers in the natural series.

In view of the great theoretical and practical importance of children understanding the principle governing the formation of higher series units, the most serious attention is paid to this at all stages of number work. Thus, when studying numbers up to 100, children become aware how 10 units join to represent a new unit of calculation, the 'ten', and that counting tens is done exactly like counting units. This creates the necessary basis for moving on to successive aggregations (hundreds, thousands and so on), and provides a clear understanding of the place-value principle in the writing of numbers.

A considerably greater role is played by theoretical generalisations in the study of the arithmetical operations. We have in mind first and foremost here the student's perception of the inter-relationship between these operations, and also the conscious application of the characteristics of those operations in justifying and using appropriate calculation algorithms.

It should be noted that experiments have shown that the overt introduction of, for example, the commutative law during the initial teaching stages does not bring the desired results: children do not doubt the correctness of the assertion made to them, but are unable to identify the right occasion for applying it. It is indeed fairly difficult to identify the steps involved in using the commutative and associative laws in an oral calculation such as $7 + (3 + 8) = 18$. It was therefore necessary to find such consequences of these laws as are in fact actually applied directly in making the calculation. In this connection, research by N A Menchinskaya has shown that junior schoolchildren grasp mathematical laws more easily if they are formulated as working rules. And so, to the sections in textbooks devoted to addition and subtraction were added such rules as adding a number to a sum and a sum to a number, subtracting a number from a sum and a sum from a number, and adding a sum to a sum and subtracting a sum from a sum. It is easy to see that these rules do not cover all possibilities; for example, they omit the adding of differences to a number, and so forth. This is because only the above-mentioned rules are used in the mass algorithms for oral and written calculations.

The overt enunciation of these rules and their conscious learning by pupils have made it possible to explain, by reference to them, the algorithms of addition and subtraction.

Similarly the working rule for multiplying a number by a sum (the distributive law) has been made the basis for explaining the multiplication of multi-digit numbers.

Algebraic material is included in the initial mathematics course in rather measured quantities. Its main purpose is to register perceived generalisations (for example, $a + B = B + a$, $aB = Ba$ etc), as a means of consolidating particular linkages under study (for example solving the simplest equations of the type $x - 3 = 7$ is useful for emphasising the linkages between the components of the functions and their results, rather than being an end in itself).

However, the introduction of algebraic material also serves more forward-looking aims, providing advance material (propaedeutic) for the subsequent study of algebra. It is precisely this approach, handled in the textbooks with possibly excessive detail, which is currently facing criticism in the pedagogical press. This criticism, however, while not rejecting the idea itself of some algebraic preparation of the ground in primary schools, points only to instances of misplaced emphasis, in which the goals of algebraic spade-work are either given precedence over the goals of generalising arithmetical content or else contradict them. An example of this is the often unjustified use of the equation method for solving written problems which are more naturally solvable arithmetically. In general, the discussion that has taken place on this subject has thrown light on a number of positive achievements in the traditional methods and has made it possible to clarify them. For example, it is now clear which types of written problem genuinely do facilitate the training of logical thinking in pupils, and which are right for putting to pupils up to and until they have a fairly developed grasp of algebra.

The picture of the mathematics course in the USSR's primary schools would be incomplete if no mention were made of the geometrical part of the curriculum. The distinguishing feature of this material is that it is directed not only towards questions concerned with the measurement of geometrical magnitudes, ie geometry is studied not only in connection with counting and calculation. Particular sections of the textbooks pursue the aim also of developing students' spatial imagination and the formation of generalised concepts of the various types of geometrical figures and of their characteristics.

The introduction of the new course of mathematics into primary schools began at the start of the 1970s after lengthy and diverse experiments in validating the new ideas and new textbooks. Considerable experience has by now been gained by working with the new curriculum, and this shows that the fundamental main lines of the reform of initial mathematics teaching were defined correctly. The new textbooks have been approved as stable, and for a number of years now they have been part of the stock of textbooks, free use of which is guaranteed by the USSR constitution.

A new stage in the development of the primary school mathematics course in our country is expected with the currently planned move to begin teaching in schools from the age of six. In accordance with the decision of the XXVIth CPSU Congress the necessary conditions should be created during the new five-year plan period for the universal teaching of children from the age of six. It is also envisaged that a five-day school week with two days off will be introduced in primary schools in place of the present six-day week. This will require increasing the length of stay of pupils in primary schools to four years while maintaining the range of coverage and the level of instruction and training.

If the strictly mathematical problems which arise in connection with this move can be said to be by and large solved, the pedagogical problems of teaching six-year-olds are at present in the forefront of minds in the pedagogical community. A number of substantial questions will need to be answered: will the move to beginning teaching children from the age of six call for radical amendment of teaching methods in 'class zero', or will a minimal adaptation of existing methods suffice? Should one use the existing system of 'zero-level' training in different types of educational establishment, or should one aim at schools only? These and many other matters will have to be cleared up by means of serious theoretical and experimental work embracing virtually all the Union Republics and all regions of our country. During the period 1980-85 there are to be prepared new teaching and methodological handbooks on the primary school mathematics course which satisfy the already existing system of criteria for methodological underpinning of the teaching process and which take account of the results of the mathematics teaching reform and of the findings of the experiments.

The efforts of numerous teams of researchers, methodological specialists and primary school teachers are currently devoted to solving these pressing problems.

At the same time more forward-looking research is being done on the methodological problems of initial mathematics teaching. Suffice it to mention here only the more substantial examples of this research work. That being undertaken by a team from the Scientific Research Institute of General and Pedagogical Psychology of the USSR Academy of Pedagogical Sciences, under the leadership of V V Davydov is devoted to a fundamentally new approach to the teaching of junior school children and is based on the formation of generalised knowledge and abilities in the field of measurement and on the subsequent formation of the concept of number. There is interesting research being done under the leadership of Corresponding Member of the USSR Academy of Science, A P Ershov, on introducing the study of information science (informatics) for junior schoolchildren. Closely related to this are the writings by Corresponding Member of the USSR Academy of Pedagogical Sciences, V G Boltyansky, on research into the possibilities of using micro-computers in teaching mathematics, starting with Class I. And finally we would mention the new conception of the primary school mathematics course developed under the leadership of Member of the USSR Academy of Pedagogical Sciences, A I Markushevich, a conception deriving from theoretical and pluralist perceptions and one which builds the remainder of the course upon them. These and other works are at present in the experimental stage and their results are unlikely to affect the content of school textbooks in the near future. Nonetheless, the very variety of the approaches outlined here is a positive factor enabling us to be confident that if schools are confronted with new tasks, the lines on which they will be solved will have been foreshadowed in these and in further pieces of research by our country's scholars and pedagogues.

In his presentation, Dr Firsov spoke of the current pressures to raise the level of arithmetical skills, and asked whether education should be directed toward utilitarian aims, or have a more fundamental character. In his opinion, the latter is the only practicable basis in a rapidly changing world. At primary level however, there is a need to provide a broad base on which secondary mathematics can be taught.

There is a concentration on basic arithmetic in the first three years: in the first year addition and subtraction of numbers less than 10; in the second year, multiplication and division of numbers less than 10; and in the third year all these operations for numbers greater than 10. There is some measurement and money in the course, a small amount of algebra to consolidate the arithmetic, and geometric shapes and measurement are included. Fractions are introduced as 'shares' but calculating fractions is left until the 4th and 5th grades. Asked if he saw any point in teaching fractions, Dr Firsov replied that without fractions you could not teach algebraic transformations, and your mathematics would be deficient for physics.

This concentration on basic arithmetic is justified because arithmetic is of value not only in itself, but as a basis for algebra and the rest of mathematics, because it helps to develop qualities of honesty, persistence and accuracy in children, and because it is unwise to make drastic changes in content with every swing of educational theory. This is not to say that there is no place for calculators in primary school, but they can be used to develop mathematical intuition rather than to replace basic arithmetical algorithms.

Premathematics has recently become an important educational issue in the USSR because children are now beginning school at 6 instead of 7 and come with less pre-mathematical knowledge. Structural apparatus is found in some schools but its provision is not compulsory. Methods include discovery by experiment. This provides greater motivation and leads on more easily to the connection with algebra.

4b THE EARLY PRIMARY YEARS (AGES 4-9)

E A Albany, Director, Nuffield Mathematics 5-11 Project

The last fifteen years or so have seen simultaneous changes in both the content and the methods involved in primary school mathematics. Carrying out these changes was further complicated by the adoption of the metric system in place of the old Imperial units.

Changes in content have sought to broaden the subject matter from arithmetic processes, involving number, money and measures only, to include much more work on shape, symmetry, the collection and display of data (pictorial representation), co-ordinates, estimation, number patterns, flow charts and elementary logic based on sorting.

Changes in method have placed much greater emphasis on 'learning by doing', making use of both 'home-made' and commercially manufactured apparatus. The importance of linking the extension of mathematical vocabulary with language development is also stressed.

Ascertaining whether or not children really understand the mathematics they are doing depends upon the level of communication between pupils and teacher. At an early age this communication is verbal and during discussion the teacher will be able to introduce and encourage the use of correct mathematical vocabulary. Every attempt is made to encourage the children to reach the stage where they are able, even in their own words, to explain what they are doing. This is seen as preferable to the reciting of a ritual learned by rote.

If children are to apply their mathematical skills to real life, they must be able to abstract the mathematics from a situation. For example, are there as many words and phrases in the Russian language as there are in English which imply subtraction? (Take away, count up to, find the difference between, how many more than?, how many fewer than?, how much change (in money) is given? remove, lose, eat, sell, decrease, etc.)

The ability to perform a calculation quickly is, in itself, not enough. The aim should be 'thoughts without tricks' rather than 'tricks without thought'.

Initially, environmental materials such as sea-shells, buttons, pebbles, beads, pencils, small plastic toys etc are used to introduce mathematical concepts. Simple, home-made apparatus (such as plug-in cards) lead to expendable worksheets or workbooks where children merely have to put a ring round a symbol, draw a line joining two pictures, or write in a number or word. Later, at about 7 years of age, children make the transition to using printed texts and emphasis is placed on careful layout of written work.

Gradually, environmental materials are replaced by more structured apparatus. This can also be made by the teacher or selected from the wide variety of manufactured materials which are available. Most schools are equipped with at least one type of structural apparatus, either rods or interlocking cubes. (Editor's note: Mr Albany's presentation was accompanied by a comprehensive display and demonstration of apparatus used in primary schools; a list of apparatus in common use is given in the Appendix.)

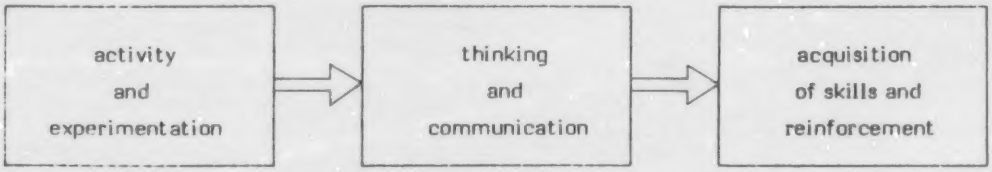
The importance of the concept of place-value is recognised by the growing tendency to introduce simple games and activities which involve the grouping and positioning of objects. The intention here is not to introduce multi-base arithmetic to young children, but rather to emphasise that the value of a numeral depends upon its position.

Failure to have a working understanding of the concept of place-value is one of the root causes of lack of numeracy. Some teachers are still relying far too heavily on an abstract approach, and far too few are familiar with, or prepared to use, the many excellent pieces of practical, structural, concrete apparatus that would be of great assistance to them with regard to this particular aspect.

For children of the 7 - 9 years age range, multibase materials such as Dienes's or Tillich's blocks reinforce the understanding of place-value using bases smaller than 10 (see Appendix).

The appeal of pattern in number work is used to facilitate the learning and retention of number facts such as addition and multiplication bonds.

The generally accepted line of development for a primary child learning mathematics is:


activity and experimentation
thinking and communication
acquisition of skills and reinforcement

APPENDIX - TYPES OF STRUCTURAL APPARATUS IN COMMON USE

Rods

Stern Structural material consists of wooden rods of $\frac{3}{4}$ inch (19 mm) square section. Each whole number between 1 and 10 is represented by a coloured rod clearly marked in unit segments. Ten colours are used so that each colour is associated with the particular number it represents. The rods are supported by an extensive range of apparatus including counting boards, pattern boards, number tracks, number cases and a box of 100 cubes of mixed colours.

A Cuisenaire set consists of wooden rods cut in lengths which vary (by 1 cm) from length 1 cm to length 10 cm, all of cross section 1 cm^2. Each of the ten different lengths is coloured one of ten different colours and this identifying association of colours and lengths is consistently applied throughout the set. These rods, which are not segmented, are used primarily as relationship material. There is a large range of materials and publications to support the use of Cuisenaire equipment, including several text books for teachers and a pupil's textbook series.

The Colour-Factor set consists of coloured wooden rods 1 cm square cut in lengths from 1 to 12 cm. Each length has a distinctive colour for quick identification and selection and all sections of the same length share the same colour. This composite material is similar to Cuisenaire but with the 11 and 12 rods.

Interlocking cubes (plastic)

Each type has its own exclusive interlocking device which forms an integral part of the cube, enabling a child to construct rods of any length, colour or combination of colours from individual cubes. The cubes can be used to investigate both collection of units (analysis) and composite entities (synthesis).

Unifix interlocking cubes are ¾ inch (19 mm) in section. They are available in ten distinct colours but the colour has no significance as a representation of value. There is a large range of apparatus available with ancillary equipment and a teacher's manual for reference.

The Centicube is a precision-made plastic centimetre cube which weighs 1g and is supplied in sets of 1,000 (200 each of five colours). The Centicube has four slots, one tongue and one blank face for marking or numbering and interlocks to illustrate number. It is versatile material, used not only as structured apparatus for number experience but also for shape and measurement in metric units.

Multilink interlocking cubes are 2cm in section. They are available in different colours but, as with Unifix and Centicubes, the colour has no value significance. Counting boards, pattern boards etc are also available as supportive apparatus.

Multibase arithmetic blocks (including Dienes's and Tillich's blocks)

These are in natural wood or in coloured plastic using the 1 cm cube as the unit. The decimal system is represented as follows: small cubes represent units; rods of 10 units ('longs') represent tens; slabs of 10 longs ('flats') represent hundreds, and large cubes of 10 flats ('blocks') represent thousands. Systems of base numbers 2, 3, 4, 5 and 6 are represented similarly. This material deals with the place-value concept and provides experience in number systems other than the decimal; this facilitates the abstraction of those mathematical and notational generalisations underlying different number systems.

Mr Albany illustrated his paper on the early primary years from the materials produced by the Nuffield Maths 5-11 Project.

In these days children know three or four times as many words as children of twenty years ago. Language is all important for communication between children and teacher. There is a tendency to think that if children can recite a row of numbers, they can count, but they may not be matching one object to one number. Verbal communication with the teacher is necessary to show what they understand. We can then start from language used by the children and attempt to refine it and make it more precise.

Simple equipment has been designed, some that the teacher can manufacture.

Storybooks were found to lack vocabulary on comparison and position, and so the Nuffield Project has provided storybooks to make good this lack. Two were shown - 'Round red apple' which distinguishes between 'round' used as an adverb and as an adjective, and 'Three buses', which introduces the words 'first', 'next' and 'last'.

BBC Schools Television: Mathematics

On the evening of Thursday, 10th September, the seminar participants enjoyed a presentation by Mr David Roseveare, the BBC's Schools Television Senior Producer for Mathematics. Mr Roseveare surveyed asurveyed a range of BBC material, at both primary and secondary levels, basing his presentation on extracts from a variety of programmes which illustrated various principles for the effective production and classroom use of mathematics television programmes. He also emphasised the importance of printed support material, for teachers and/or for pupils, and of careful preparation by the teacher, if the maximum value is to be derived from the use of television in the learning of mathematics at school.

6a MATHEMATICS IN THE MIDDLE YEARS (AGES 9 - 13)

Dr A W Bell, Shell Centre for Mathematical Education, University of Nottingham

Although 'middle schools' for pupils of 9-13 or 10-14 exist in some areas, the general pattern still consists of primary schools (5-11) and secondary schools (11-18), and the issues of current concern tend to be different in the two types of school. We shall therefore treat them separately in most of this paper.

Primary mathematics

In the field of primary mathematics the issues of greatest interest are probably the evolution of the curriculum, and the attempts to evaluate the relationship between teaching styles (teacher-or child-centred) and learning outcomes.

The changes in curriculum will be evident throughout this section; discussion will focus first on the comparative researches, then on the more detailed recent surveys made by the Government's Assessment of Performance Unit (APU), and finally on the most important feature of curriculum change in the immediate future, the advent of the pocket electronic calculator.

Comparative research

It is perhaps important to begin with a brief historical reminder. In 1862 the mathematics syllabus for the elementary school consisted entirely of a sequence of increasingly complex computations with number, money and measures. At ages 5 and 6, numbers up to 20 were to be recognised, and the addition and multiplication tables known. Then the four operations were learnt, first for numbers, then in money, weights and measures. The Education Act of 1902, followed by the 1905 'Handbook of suggestions to teachers', promoted a widening of the syllabus, with more emphasis on utility, and in the schools for older pupils, algebra, trigonometry and mechanics came gradually into the curriculum. In the primary schools there was a slow change towards less complex computation and more attention to conceptual understanding and to applications, until, with the general abandonment of the 11+ selection test for secondary schools in the 1960s, more rapid development began.

In these more recent times, there have been three main influences on the development of mathematics in the primary schools; these are firstly educational theories emphasising child-centredness and learning from the environment, stemming from Rousseau, Pestalozzi, Dewey and others, secondly the insights into children's developing understanding of number and concepts of measurement exposed by Piaget, and the related development of structured materials and associated learning theories, by Stern, Dienes, Cuisenaire and Gattegno, and thirdly the 'modern mathematics' movement, originating with the Bourbaki mathematicians in France.

A comparative survey of eighty-seven primary schools made in 1959 (Biggs, 1967) classified their mathematics teaching as traditional, structural or motivational. 'Traditional' emphasised direct instruction in the computational processes, with practice; 'structural' used Stern, Cuisenaire or Dienes material for learning number facts and relationships; 'motivational' included a substantial amount of practical activity with environmental materials or situations, for example, timing another pupil running four laps round the playground, measuring the distance and calculating the speeds, and perhaps plotting a graph to show the changes in speed over the four laps. The tests used included computation, verbal problems involving choice of operation and a computation, and a 'concept test' including questions on place-value and choice of operation without calculation. The results showed differences attributable to the different methods which were highly significant statistically (levels from 0.1% to 5%), though small in absolute value (1-3 scale points in 100). The motivational group did noticeably less well than the other groups on all the tests; the traditional group did slightly better than the structural group on computation, and vice versa on verbal problems. These results perhaps indicate that the environmental activities do not promote the development of computational skill (this was the area of greatest difference).

These results might be regarded as predictable, in that both traditional and structural methods place stronger emphasis on pure number relationships and skills than the motivational; the environmental activities of the latter method are mainly directed at showing the relevance of mathematics to everyday situations and promoting the recognition of appropriate mathematical concepts in these situations. The amount of computational practice would be less in this method, and, although one might expect that verbal problem solving would be beneficially affected, one could not expect this effect to be as strong as that obtained from a diet of more intensive practice of such verbal problems themselves.

A study undertaken some ten years later, around **1970**, used only three schools, one traditional, one modern or 'Nuffield' maths, and one school which claimed to 'keep a balance' between the new and the old. (The Nuffield mathematics project combined the environmental child-centred discovery approach with a much stronger emphasis on relational understanding of number and some elements of modern structural mathematics.) A wider range of tests was used, including some similar to those of the previous study, and others which tested divergent thinking. The results on the main mathematical tests were much the same as before, but the modern methods did show up well, and the traditional methods poorly, on some of the divergent thinking tests (Richards and Bolton, 1971).

A broader and more descriptive survey of primary school mathematics was made during the years 1972 -75 (Ward, 1979). The main findings of this survey have been superseded by a later work, but one of its interesting features was that it recorded teachers' views of the state of primary school mathematics. Asked for their reservations, 29% expressed concern at the 'neglect of basic (computational) processes', 12% at the 'lack of structure or progression' and 11% at 'over-full syllabuses'. To the question about strengths, 46% mentioned 'increased understanding of concepts', 24% 'enjoyment and interest', 20% 'practical activities' and 16% 'relationship with child's experience'. It is clear that these teachers are by no means the passive surveyors of a curriculum decided by others, but involved and self-critical professionals who are able to exercise considerable choice in the content and methods of their teaching, and feel a considerable responsibility in regard to these choices.

There are two recent studies which attempt to relate teaching styles and pupil attainment across the primary curriculum. The earlier of these categorised teachers on the informal-formal dimension on the basis of their replies to a questionnaire which asked whether or not they taught pupils as a whole class, had desks arranged in rows, gave weekly tests, and so on. On standard tests of reading and mathematics, the pupils of the more 'formal' teachers were more successful, although the most successful teacher of all was an informal one (Bennet, 1976). The later study was more thorough, basing its categorisations of teaching style on actual observation of classrooms followed by cluster analysis of the resulting data (Galton and Simon, 1980; Galton, Simon and Croll, 1980). This yielded four main styles of teacher. The 'individual monitors' (22% of the sample) moved rapidly from table to table, checking work and giving brief instructions to pupils working mainly on individual tasks. The 'class enquirers' (16% of the sample) introduced topics to the whole class, with questioning, using more talk at higher cognitive levels than the remaining groups. The 'group instructors' (12% of the sample) set tasks to groups of pupils, giving sufficient information to structure the work of the group before leaving them to discuss amongst themselves for a while. The 'style changers' (50% of the sample) who used a mixture of the previously described styles, comprised three sub-groups: the infrequent changers (10%) made planned changes in organisation to correspond to the pupils' developing ability to work on their own; the rotating changers (16%) used different styles for different curricular activities and pupils changed from one to another of these at specified times of the day; the habitual changers (24%) made regular but unplanned changes of organisation - generally from individual work to class teaching as the level of noise and unrest in the classroom became too high.

A corresponding categorisation, by clustering, of pupil working styles was also made, the types being named attention seekers (20%), intermittent workers (36%), solitary workers (33%) and quiet collaborators (12%).

Progress in mathematics was measured by modified versions of the Richmond Tests of Basic Skills which contained a greater variety of material than those used in the earlier studies. Judged by the progress of pupils on these tests, the order of success among the teaching styles was as follows: the class inquirers did best; next came the group instructors and the infrequent changers; least successful were the rotating and habitual changers and the individual monitors. Thus, once more we have the result, consistent with the earlier studies, that a whole-class teaching method proves the most successful. We also have the observation that this method contained the greatest amount of teacher-pupil interaction concerning ideas and problems as distinct from brief managerial or factual statements. The poor performance of the individual monitors, whose style contained a high proportion of low-level interactions, reinforces the suggestion that this is a significant factor. However, these results are somewhat modified when the pupils' progress in a broader range of study skills is taken into account. Overall, the infrequent changers proved the most successful; they were those who were able to use effectively both class and group work.

The APU survey

One of the most significant developments on the national educational scene in the last few years has been the establishment of the APU (or Assessment of Performance Unit) of the Department of Education and Science. This arose partly because of the embarassment of the government in the face of criticism from industry and elsewhere of the achievements of school leavers, since there were no data in existence from which the government could either confirm or refute the allegations. There was also some concern regarding the achievement of immigrant groups, on which again the government wished to have some reliable information.

After a lengthy period of the development of test instruments, the APU has now conducted two annual national surveys of the mathematics attainments of a substantial sample of pupils aged 11 and 15. Similar tests in English and science are to follow, one and two years later respectively than the mathematics. The present plan is to conduct five annual surveys at each age, followed by a programme of testing at less frequent intervals. It is convenient to describe here some aspects of the reactions both to these surveys and to the design of the test material, and the results. This will provide information regarding the normal content of mathematics teaching in this country at and around the age of 11. The curriculum framework used for the APU tests at the age of 11 is shown in the figure below.

The curriculum framework: main categories of content and outcome

Main content category	Type of performance or outcome	Concepts	Skills	Applications	Generalisation and proof	Investigations and creativity	Attitudes	
Measures		Money, time, mass, temperature					Liking, difficulty and utility of mathematics	Liking and difficulty of mathematical topics in the 13 sub-categories
		Length, area, volume, capacity						
Geometry		Shapes, lines, angles						
		Symmetry, transformations, co-ordinates						
Number		Concepts: whole number	Computation: whole numbers and decimals	Applications of number				
		Concepts: decimals and fractions	Computation: fractions					
		Ratio and rate						
Algebra		Generalised arithmetic						
		Sets and relations						
Probability and statistics		Probability and data representation						

The APU policy is that the content of the tests should reflect the diversity of the mathematics curriculum, both with regard to the range of content taught and through the variety of mathematical activity in the school. Thus there is a two-way categorisation: by content, and by the type of task. These learning types include concepts, skills, applications, generalisation and proof, investigations and attitudes to mathematics. In all, there were about thirteen sub-categories, each represented by about fifty-test items (the total sample comprised some 15,000 pupils. Each pupil took a fifty minute test comprising items drawn from three sub-categories. Individual schools and pupils are not identified. A smaller sample of about 1,000 pupils took practical tests administered by visiting testers in one-to-one interviews).

Full details of this first APU Primary Survey may be found in the report (Foxman, 1980). The report quotes for each category the mean score and the distribution of scores, and also reports the responses to a teacher questionnaire asking for teachers' views on the appropriateness, for their pupils, of the items in the test. But perhaps the most useful aspect of the report for the teaching profession is the provision of clusters of closely related items with their facility values.

The most distinctive feature of this APU testing is probably the practical element. Twelve short practical exercises were used; six of them are reported fully in the report. The rationale for the inclusion of these practical tests was twofold. First it was argued that since a considerable amount of practical work, including the use of a variety of forms of structured apparatus and also of environmental situations, was in use in the schools, it should be represented in the testing. Secondly, and perhaps more importantly, it was felt that an interview test provided a much more realistic situation, much more like those in which the pupil was likely to require to use his mathematical knowledge, than the written test. Undoubtedly both the testers and teachers at large have expressed a warm welcome for the inclusion of this aspect in the testing. The tester was instructed to supply prompts and if necessary explanations in the course of the interview, and to note such events and the salient features of the pupil's response on his record sheet.

The survey also included the use of a number of attitude questionnaires. The chief of these was a Likert scale containing three sub-scales, one for liking of mathematics, one for perceived difficulty of mathematics and one for the usefulness of the subject. On the liking scale, out of a possible score range of 0 to 34, the mean was 20.5 and standard deviation 8.8, indicating a broad spread of response with a generally favourable bias. The mean score on the difficulty scale was close to the middle of the range, and the standard deviation was somewhat less than that of the liking scale. The utility scale had a mean score of 20 out of a possible range of 0 to 24, showing generally strong agreement with the statements concerning the usefulness of mathematics. Some pupils were also asked to rate the mathematics of the different sub-categories for liking and difficulty. Here, the number computations were both liked and regarded as easy, while within the geometry catetories, the modern topics such as symmetry and tessellations appeared to be liked less and to be thought more difficult than the more traditional topics such as shapes and angles. However it is not clear whether in this case the pupils were responding to the images conjured by the words themselves rather than to the actual mathematical content.

The general conclusion from the survey as a whole was that 'most eleven year olds can do mathematics involving the more fundamental concepts and skills to which they have been introduced and also simple applications of them ... There is however a fairly sharp decline in performance as pupils' understanding of the concepts is probed more deeply and as their basic knowledge has to be applied in more complex settings or unfamiliar contexts'. A typical example of this is that the reflection of a simple shape in a vertical mirror was drawn successfully by 80%, but only by about 20% if the mirror was diagonal. Another example is that the division 84 ÷ 4, presented thus, was answered correctly by 70%, while the same calculation offered in the form $\frac{84}{4}$, obtained a facility of only 40%.

When the idea of a national survey of mathematical attainment was first publicly discussed, considerable apprehension was expressed by the teaching profession on account of fears that the results would lend themselves to simplistic press reporting and further uninformed criticism of the educational profession. There was also apprehension about the extent to which the material being tested would be perceived by many teachers as that on which teaching effort should be concentrated, thus leading to a narrowing of the curriculum of the schools. In the event, the second danger has been largely averted by the breadth of coverage of the tests. However, there remains the possibility that the higher-level and more intangible aims of mathematics teaching are becoming less highly valued than the kind of straightforward knowledge of skills and concepts which is relatively easily tested. Regarding the first problem, press criticism seems to have died down somewhat in the last year and we must hope for constructive and positive responses when the currently sitting Committee of Inquiry presents its report later this year. What has been disappointing is the rather low level of interest shown by teachers in the results of the APU tests.

The calculator

The most significant curriculum change in the primary school in the next ten years is likely to be that resulting from the gradual assimilation of the pocket electronic calculator. Already a high proportion of pupils either have their own machine or have one at home and use it for doing or checking their homework. But most teachers remain uncertain of how to use it in the mathematics curriculum, and many are anxious lest pupils should become mathematically 'soft' and unable to calculate. The Shell Centre for Mathematical Education at Nottingham University has run for three years now a project in which ideas and materials have been developed and circulated among a group of about fifteen primary schools. A similar project in secondary schools is operating in the Durham area. The report of the pilot stage of the former project (Bell et al, 1978) found that the calculator encouraged and helped children in developing their number skills; this accords with American findings. It provided the basis of some stimulating number investigations, which both reinforced the memory of basic facts and revealed new number relationships, and it gave a means of checking the results of practice calculations and exposing misconceptions in a non-threatening way. In topics like decimals particularly it provided a 'test-bed' on which predictions could be checked and the outcome used as a reference point for discussions between teacher and pupil, making for much more fruitful interactions than when the teacher has also to be the arbiter of correctness of the results. Realistic applications of number also become possible, and as the burden of calculation is removed, more attention can be given to the mathematical principles. A particularly successful game has been 'Calculator golf', in which two players each select two single digits (their 'clubs'). Then one of them decides on a distance (< 100), and each attempts to attain this distance by using his two digits as few times as possible.

Changes in the curriculum which appear likely as a result of the use of calculators are a reduction in emphasis on long division and long multiplication, and increased work on methods of checking calculations (eg by reversal), on estimating answers, and on mental calculation (eg 500 x 6). The calculator has also incidentally helped to expose weaknesses in the choice of operations; in traditional verbal problems, with numbers chosen conveniently for the subsequent calculations, the correct operation can often be identified from incidental details without recognition of the true operational structure (Bell, Swan and Taylor, 1981).

Secondary mathematics, 11 - 13

Significant events of the last ten years relating to this phase of education have been firstly the spread of 'modern' courses, followed by critical appraisals (and even attacks), and the development of courses combining both traditional and modern material, secondly the development of individualised work card systems in response to the demands of teaching mixed ability classes, thirdly greater emphasis on the learning of process aspects of mathematics, through the introduction of more extended individual mathematical investigations into the curriculum (a minority enthusiasm), and fourthly the emergence of survey results showing fairly wide discrepancies between syllabus expectations and actual pupil attainment.

Modern mathematics

The brand of modern mathematics which has taken root in Britain is typically represented by the School Mathematics Project (SMP) course. (This course is now followed by more than half the secondary schools in England.) The innovations include some aspects of algebraic structure, some elements of set concepts (union and intersection, and inclusion), arrow diagrams for relations, modern notation for functions, a more explicit treatment of the commutative, associative and distributive laws, and some use of matrices and vectors. However, this material is dealt with informally, not axiomatically. In geometry there is an abandonment of the axiomatic structure of Euclid as well as a substantial change in content, with the hierarchy of transformations of Klein's Erlanger program now central. Reflection, rotation and translation are studied first as separate concepts and later combined; shears, enlargements and informal topological transformations are also considered, and are represented by matrices. There is some extension of descriptive three-dimensional geometry, and trigonometry is retained. Binary and other number scales are introduced, and so are elementary statistical ideas. The time for the new work has been found by reducing the amount of manipulative practice (Thwaites, 1972).

This refers to the SMP syllabus for the O level (16-year-old) examination, as introduced in 1961. However, by about 1974 it was generally felt that the reduction of the amount of practice had been a mistake, and sets of supplementary exercises, covering all parts of the course, were published.

During the years 1973-77 a Schools Council project entitled 'The mathematics curriculum: a critical review' was in operation at Nottingham. This was motivated by the feeling that the mathematics teacher by then 'faced a daunting array of novel classroom material, and would welcome a basis for constructive and critical discussion of the curriculum'. Working groups of teachers contributed views and material, which were then edited. The booklets on the major mathematical topics each contained extensive and detailed reviews of a range of currently available school texts, commentaries on methods of treating both old and new topics, and expositions of possible new material, some of it aimed at giving the teacher a more advanced standpoint from which to view the existing material. They also contained the material thought to be essential for the average pupil.

Individualised learning systems

One of the issues under active discussion in recent years has been that of mixed ability teaching. With the development of comprehensive secondary schools, some Heads and staffs have felt it inappropriate to segregate pupils by ability immediately on entry to the school. Indeed, some have felt that the full benefits of comprehensive education in giving the pupils of various abilities the experience of being part of a single community entailed that they should have this experience within their teaching groups. The changes in teaching style required when the whole ability range is contained within a single teaching group have presented teachers with an unfamiliar demand. Teachers of English and of the humanities in general have not found it difficult to conduct their teaching in such a way as to provide meaningful experiences in the same classroom for pupils of this variety of abilities, but in mathematics and in modern languages it has been found particularly difficult, and teachers of these subjects have found it necessary to make radical modifications of their methods of teaching and of classroom organisation. A quite common solution in mathematics has been to abandon class teaching of the traditional form, in which the teacher addresses the class and sets a common set of exercises to all pupils, and to adopt an individualised system. (To keep this issue in perspective it should be noted that although it has been much discussed, in fact only 46% of comprehensive schools teach mathematics to mixed ability classes in the first year and only 19% and 7% continue it into the 2nd and 3rd years respectively. At the same time, only 16% of comprehensives used individualised methods exclusively in the first year, and a further 16% used them in conjunction with other methods.) (HMI, 1979; see also Schools Council, 1977.) In the typical individualised system, a set of separate work cards containing the mathematical tasks is kept in a box in the classroom, and pupils either choose, or are assigned individually, the cards at which they are to work. They then proceed at their own pace through the cards on each topic, finishing each topic set of five or six cards with a revision card and an individual test. This is usually taken to the teacher for marking and the pupil is then directed to a new topic. Most teachers and most pupils seem to like these systems, but with reservations. The pupils in general like being able to get on with their work without having to spend a long time listening to the teacher, while teachers find such classes better behaved and easier to manage. Very little formal comparison of attainment under these systems has been made, and it would appear that differences in attainment are slight, being much less, for example, than those attributable to differences between teachers or schools. The chief criticisms of work cards have been of their inability to communicate a meaningful problem to a pupil. This arises from the need for the card to be intelligible even to pupils of low ability, and to present a task through which the great majority of pupils can progress smoothly, without requiring help from the teacher. Thus, in some cases the tasks are performable by the use of mathematically irrelevant cues. For example, the blanks in the following question can be filled by continuing the patterns along the rows of numerators and denominators and without any appreciation of the principles of equivalence of fractions.

$$\frac{1}{2} = \frac{\ }{4} = \frac{3}{\ } = \frac{\ }{8} = \frac{\ }{\ }$$

On the other hand, having the entire set of material for the year in use all the time in the classroom means that it is never possible for the teacher to have a discussion with the class as a whole, since, for any chosen topic, there will be pupils in the class who have completed the work on it and others who have not yet begun. Some teachers have attempted a partial solution to this problem by saving certain topics to be dealt with by class teaching.

The alternative approach is to retain the class as a unit in the sense that all pupils are working on the same topic at the same time, but to provide for different levels of ability and speeds of working within a single topic. Textbooks have always had to attempt to cope with this problem to a smaller or greater extent and they have generally done this by ensuring that later chapters on a given topic have a reasonable amount of overlap in content with earlier chapters. The faster pupils will get through the easier work more quickly and thus still have most of their time available for working on the harder exercises on the topic. A development of this approach has been adopted by one recent scheme, in which the abler pupils progress to higher levels of generalisation and proof, and extend the starting problems in consultation with the teacher. (Bell, Wigley and Rooke, 1978-79).

In a number of schools the introduction of mixed ability teaching has been the stimulus for the mathematics department to design and write its own teaching materials. This is of course a task of considerable complexity, calling for powers of organisation and of leadership on the part of the head of department. Sometimes the results have not been of high quality, but in other cases the enterprise has led on to commercial publication. But in all cases the degree of insight gained by the members of the department both into the mathematics curriculum and its structure, and also into problems of teaching and learning, has been considerable.

Some departments run a more elaborate workshop approach to mathematics, in which, particularly during the years up to 13, a variety of short topic books, problem cards and practical materials is available, from which the children may choose under guidance their tasks.

A very general problem of mathematics teaching which is thrown into relief by the attempt to teach mixed ability classes is of course that of matching the particular task to the pupil, that is of ensuring that the task is intelligible to the pupil and within his comprehension, but at the same time extends that understanding by a significant amount. A substantial research study on this problem, in which the match of pupil and task is being compared in seven classes of pupils from 11 to 12 years, covering a number of different types of class teaching and word card systems, is currently in progress at the University of Nottingham.

Process aspects of mathematics

The essence of mathematics is the abstraction of the relational content of a situation and the manipulation of these relationships to obtain some new insight; usually this also involves the representation of the relation-ships by symbols or diagrams. (I think that Alexandrov (1963) and Krutetskii (1976) would agree.) It is therefore important that pupils should have experience of this full creative mathematical process, and not just perform exercises or solve problems aimed at improving their understanding of some known mathematical relationship or process. The Association of Teachers of Mathematics has been particularly concerned, since its foundation in 1952, to share ideas and materials which take account of this viewpoint (see Fletcher (1964), ATM (1967) and the quarterly journal 'Mathematics teaching'), and my own research has been in this area (Bell, 1976, 1979).

An example of a pupil's extended investigation is 'The remainder problem'. It was done by a 13-year-old girl, a pupil of a colleague of mine, Mr Eric Love, at Wyndham School in Cumbria. The problem is:

> A boy counts his sweets in fours, and has two left over;
> when he counts them in fives, he has one left over.
> How many sweets did he have?

This pupil's approach to the problem was as follows (see also Appendix A):

> 'I thought about this problem and wrote it out like this: 4r2, 5r1.' (An interesting little bit of symbolism.) 'I got the answers 6, 26, 46, 66, 86 and so on.'

Already this girl has generalised the problem from what was initially given to her. The answer 6 could have been the end of the story, but she looked for other solutions and has found a whole set, and has discovered the relationship connecting the differences in this sequence with the divisors, 4 and 5, given in the problem. Next she generalises the problem, considering (3r2, 4r2) and later (4r2, 6r4), which lead her to grapple with, but not quite grasp, the concepts of highest common factor (HCF) and lowest common multiple (LCM).

> 'I ended up with, if the numbers have no connection, they go up in multiples of the two numbers, and if one number is a multiple of the second the numbers go up in the largest number.'

After a further generalisation, to problems with three conditions, she takes up the question of how to find the starting number. She is unable to find an explicit solution, but gives an effective algorithm, based on constructing a table.

This is an example of an open investigation of which the main interest is in the processes of generalisation, abstraction, representation and proof. But it also offers an opportunity for taking up and teaching the concepts of HCF and LCM and their properties, in a situation where the need for them can be clearly seen. A varied collection of starting points for investigations has been published by the ATM (1980).

One English school has arranged, with one of the Examination Boards, a special O level examination in which 40% of the assessment is based on a number of mathematical investigations, done during the course and submitted by the pupil. Others have similar arrangements for the CSE examinations.

Results of survey research

The experiments of Piaget which outline stages in the understanding of mathematical concepts have been extended by other workers into more areas of the curriculum. In particular, a project operating during 1973-78 at Chelsea College aimed to produce a 'conceptual map' of the secondary school mathematics curriculum. By interviews and then by tests of stratified samples of several hundred pupils, levels of difficulty were established in fractions, place-value and decimals, choice of operation, ratio, reflection and rotation, and several other topics. Usually pupils in the 2nd, 3rd and 4th year of secondary school were tested. The results show a number of surprises and an extensive degree of mismatch between curriculum expectations and attainment. For example, only a minority of pupils appear to acquire a real understanding of decimal place-value beyond the first decimal place, and the identification of the operation required for problems such as the following was only successfully made by, on average, half the pupils:

The price of minced beef is shown as 88.2 pence for each kilogram. What is the cost of a packet containing 0.58 kg of minced beef?	88.2 + 0.58	0.58 - 88.2
	88.2 - 0.58	0.58 ÷ 88.2
	88.2 ÷ 0.58	0.58 x 88.2

Similarly, while most pupils could construct the reflection of a simple figure in a vertical mirror, to do this in a sloping mirror was a minority accomplishment (Hart 1981). The APU surveys reinforce these indications (Foxman et al, 1980). Perhaps more significantly, the amount of progress from year to year in the depth of understanding of these key concepts was relatively small, of the order of 10% per year. It thus appears that while teachers in general are teaching a considerable number of particular methods and concepts week by week, most of this learning is short-li[illegible] and the residual growth of understanding is slow. This suggests that teaching might profitably aim more directly to promote this growth of understanding. Current research at Nottingham on teaching methods is aimed in this direction (Bell et al, 1980).

A note on curriculum consensus

Since Britain is perhaps the only country in the world in which the curriculum is not yet presecribed by any authority outside the school, it may be of interest to comment on the processes by which such uniformity as exists arises. For primary schools, some authorities do publish guidelines for mathematics. Such sets of guidelines are usually drawn up by committees involving teachers and local authority advisers, and in general, schools are at liberty to use them to whatever extent they wish. In the typical primary school the mathematical programme will be designed by the staff acting corporately under the leadership of the head teacher. Sometimes schools choose to run their work for a term around some particular theme of interest, and in such cases the mathematical applications would be designed to fit in with this theme. Another process tending towards conformity is that meetings and mutual visits will often be arranged between the primary school staffs and the secondary school staffs, particularly the heads of department of those secondary schools to which the primary schools send their pupils. There are also courses and meetings run either by local authorities, by teacher-training institutions, or by local associations of mathematics teachers, at which ideas and practices are shared around local groups. In the secondary schools there is of course also the influence of the external examinations taken by most pupils at the age of 16. Although there is some variation among the syllabuses for these exams between different regions and different examination boards, this is undoubtedly a constraint which is external to the individual school. Textbooks also represent a constraint, but there is in fact a greater variety of these available than is taken up on any substantial scale. A few texts are very popular while a large number of others are used each in a very small number of schools. It is still true that within the secondary school it is largely the head of the mathematics department who is responsible for the mathematics programme of the school, and in the primary school the corresponding responsibility rests upon the head teacher.

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In his presentation Dr Bell underlined the main points of his paper and drew out the major findings of recent research. Discussion centred around the work of the Assessment of Performance Unit, which he forecast would become a determinant of future trends in mathematical education, and the status of modern mathematics.

The APU was charged with responsibility to test what is being taught. They looked at the tests already in use in schools; they did not draw up a list of topics they thought should be taught. Statistical methods were applied to the results.

Modern mathematics has not been abandoned but some courses, eg SMP, have had supplementary material added to improve manipulative skills. Recent texts have shown a greater balance between 'modern' and 'traditional' mathematics.

6b THE TEACHING OF MATHEMATICS IN CLASSES 4-8 IN SOVIET SECONDARY SCHOOLS

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It is evident that there is great interest in considering this stage of the teaching of mathematics. It is precisely in these classes that the pupils receive most of their mathematical information; here the basic algorithms are formed, and the most important skills and practices are acquired.

Giving an overall description of teaching at this stage, we consider that it is at this point that the formation of the concept of real numbers is completed, the basic content of school algebra is introduced (transformation, equations and inequalities, function), the whole of plane geometry is studied, programming is started. The content of subsequent study in Classes 9-10 is just some further algebra, clearly not playing the central role, as then most time is given to the study of the elements of differential and integral calculus and solid geometry. Therefore the teaching of mathematics in Classes 4-8 is the nucleus of school mathematical education: the mathematical and methodic decisions taken here define the whole nature of the mathematical preparation of the pupils, and the level of mastery of the basic theoretical material of the school mathematics course.

Serious pedagogic problems arising in this way can be adequately solved only in the framework of sufficiently advanced and systematised courses, which a specialist must introduce. Therefore, beginning with Class 4 (age 10), the teaching of mathematics in the schools of the USSR is undertaken by teachers who have received, as a rule, a special mathematical and pedagogic preparation in an institute of higher education. The systematised exposition of algebra and geometry, having a very specific character, demand that teaching be divided into two separate, though connected, courses.

Together with this, the speedy change to a systematic exposition, for instance, of geometry immediately after primary school, as witness the findings of pedagogic psychology, is not possible: ten-year-old children are still not able to assimilate abstract constructions which are characteristic of systematic courses; the level of their development in logic is inadequate. Therefore the structure of the subject 'mathematics' in the given stage of teaching is in two phases: in Classes 4-5 a transitional course of mathematics is taught, building on the general material of instruction in primary school and preparing for the following teaching of systematic courses; in Classes 6-8 two independent courses are taught, 'algebra' and 'geometry'. This structure is sufficiently flexible and is geared to the age of the pupils.

For the following characterisation of the mathematics course we need that which was introduced into pedagogy in the works of V P Goncharov, and developed in the works of S I Shvartsburd, the concept of the content-methodic line of the course, uniting in itself the closely linked concepts, ideas and facts of the mathematics course.

In the mathematics course in Classes 4-5 it is possible to separate three main lines, around which the basic subject matter is grouped; the remainder serves as accompanying, connecting and 'story', which remind us of these main lines. These lines can be conditionally given the following names: 'numbers', 'equations' and 'geometric constructions'.

In the course information is systematised and summarised on natural numbers, carried over from primary school. There is a further extension to studies of fractions and negative numbers. This manner of introducing rational numbers is dictated by the effort to widen, in the first place, the possibility of measuring magnitude, essential to the teaching of allied subjects. In addition to this the study of fractions leads in succession to the initial exposition of fractions, decimal fractions and ordinary fractions. The choice of such a succession creates a sufficiently natural precondition for the summarisation of algorithms, of arithmetical operations over natural numbers and decimal fractions, and is useful from the point of view of application. On the other hand ordinary fractions and negative numbers are also incorporated into the course of algebra, which greatly helps the continuity of the course.

The complexity of the textual problems now presented is sufficient for a full use of the method of equations. The solving of linear equations with coefficients from an already considered number set, created now not on the basis of the use of dependence between components and the results of operations, but by an ordinary algebraic method, penetrates the whole content of the course.

The algebraic content of the course is not exhausted by the solution of equations. The use of letters if positively helpful at the stage of summarisation and systematisation of the properties of numbers. More than this, the course of mathematics in Classes 4-5 tackles the pedagogically complex problem of the transition from the use of a letter as a symbol for a number to its use as a symbol for a variable. It is natural that this issue is closely linked with the introduction of the concept of 'algebraic expression' and with propaedeutics in its basis of understanding function.

The geometric material of the course has, on the whole, a propaedeutic character. Its central pivot is doing geometric constructions with the help of various drawing instruments, which must lay the foundation of geometric intuition, necessary for the teaching of a systematic course of geometry beginning in Class 6. Besides acquaintance with fundamental geometric figures, it is envisaged that the use of these simple transformations will give students familarity with symmetry of figures relative to a point and straight line. Finally, with the aim of training pupils to assimilate a deductive course of geometry, a definite logical training is introduced.

In Classes 4-5 the development of logical thought is linked with the mainly inductive character of the exposition of mathematical material. Deductive elements are gradually introduced. Pupils must learn to draw inductive conclusions (summaries) on the basis of the examination of particular examples, to apply general principles (definitions, rules) to concrete situations, to make the simplest one or two steps towards inductive reasoning, and to make simple classifications. Demands on the development of mathematical speech are made by the correct use of parenthetic terms, the ability to use symbolic writing (in particular the ability to write a formula in a conventional formulation of the rule or law), the understanding by the pupil of the meaning of specific logical turns of speech of the type 'if and only if', and so on.

In the algebra course for Classes 6-8 it is possible to isolate four basic threads: 'numbers', 'algebraic expression', 'equation and inequality' and 'function'. From the aforesaid it should not be understood that the corresponding material is taught in succession; on the contrary, these threads infiltrate the whole content of the course, in practice finding expression in every topic.

The number line contains in itself the systematisation of a knowledge of rational numbers in introducing irrational numbers. A significant application of the number line is the acquaintance by the pupil of the use of approximate calculations based on the properties of inequalities. Traditionally significance is given to work with the slide rule as the most accessible instrument for calculations, but the amount of use reported now has decreased in connection with the appearance of calculators. Finally the course of algebra in the eight-year school is completed with a short introduction to computer programming, which forsees an acquaintance with the writing of the simplest computer algorithms in an algorithmic language.

A significant place in the course is occupied by the training in the transformation of algebraic formulae - at first integral and then rational. In comparison with the traditional course, the amount of work with irrationals is severely diminished. This point, in general, shows a certain defect in the traditional structure of mathematics, taking much time without clearly visible worthwhile results. Therefore an attempt is now being made (though still far from being realised in schools) to search for new mathematical ways of crystallising the concept of signed exponent, avoiding the traditional scheme 'natural - zero - negative whole - rational index - concept of irrational index'. The use of fundamental skills for doing algebraic tranformations is distributed throughout the course, and so realises the aim of increasing time for bringing skills to the level of being automatic.

'Equation and inequality' is introduced into the course descriptively as a statement with variables. Algorithms for solving linear and quadratic equations and systems of linear inequalities are developed. More complex kinds of equations and inequalities are introduced.

In studying equations and especially inequalities, graphic exposition plays a significant role, connected with the study of the behaviour of the graphs of the corresponding function. It is now acknowledged that this functional approach, instead of the traditional algorithmic method, helps a more conscious mastery of the solution of equations and inequalities.

Functions are one of the central objects of study. Fundamental functions - linear, quadratic, powers, modulus, logarithmic - are studied by elementary methods with an exposition of their properties. The corresponding thread, which began back in the mathematics course for Classes 4-5 and is clearly linked with the course under consideration, is designed both for the proper provision of further algebra and for preparation for the study of the elements of mathematical analysis in the senior classes.

In describing the course of algebra for the eight-year school, one must not fail to mention the solving of textbook problems, which traditionally make up its most notable part. In Soviet methodics of teaching mathematics it is generally agreed that the solving of textbook problems, by the method of constructing mathematical models of the situation being studied and described in the statement of the problem, is a necessary and important element in teaching mathematics, in so far as it represents the application of mathematics to the solution of practical problems in the most complete form appearing in the course. Therefore in solving the textbook problems in the courses of algebra, geometry and finally mathematical analysis in the higher classes, the applied direction of the teaching of mathematics is shown, that is the making of a purposeful, full and methodological link between teaching and life, the practical activity of people.

Naturally the applied direction of teaching is not only seen in this way. Soviet pedagogues attach no less significance, for instance, to the methodologically correct formulation of the basic concepts of the course, which must arise out of practical problems and illustrate by use of them the processes of abstraction and generalisation. It is worthy of note that in the achievement of the aim of directing the course towards applications there are still significant reservations, and the efforts of methodics are concentrated on solving the problems that arise here.

Parallel with the course of algebra there is a course of geometry, which is considerably modernised in comparison with the textbook which is widely known in our country by the prominent Soviet pedagogue-mathematician A P Kiselov, from which textbook many generations of Soviet people have studied. The most essential elements of the modernised course have been worked out under the direction of Academician A N Kolmogorov. They include a strict deductive exposition of geometric transformations, with the basic object of studying the nature of proof.

The graceful attempt of A N Kolmogorov to build an essentially new school geometry, as this is now conceived, is far from being totally successful. There are here two issues of the greatest importance which are not yet fully resolved. First of all doubt arises as to the possibility of using geometric transformations from the very beginning of the systematic course of geometry because of the complication of presenting the steps of the proof of a concrete transformation to a Class 6 pupil. Secondly work is not finished on the development of a system of geometric problems which fulfils the natural pedagogic demands of accessibility, gradualness and the use of algorithms.

Therefore in textbooks we have had to avoid constructing a course purely on the basis of transformations, and have had to use the traditional methods of proof based on the congruence of triangles. In this way a definite flowing together of more contemporary and traditional approaches took place, and in the stages of development of the textbook, movement in the direction of the traditional course became more noticeable.

Alternative variants of geometry textbooks on the contrary move away from traditional approaches. However the authors of alternative textbooks acknowledge the necessity of using geometric transformations at a later stage, when a sufficiently rich store of geometric ideas will have been formed in the pupils. In the question of the deductive construction of the course all authors stand together. So, in the words of A V Pogorelov, the author of one of the most promising variants of the geometry textbook for general schools, 'the main task of geometric education in school is to teach the pupil to think logically, to argue his convictions, to prove'.

The scope of geometric material to be taught in school is now sufficiently exactly defined and will not be reconsidered. Traditional work on the geometry of polygons and circumferences is supplemented by the study of elements of analytical symmetry about a point and a straight line, the parallel turn and similarity. Metric material is closely connected with the study of elementary trigonometry. Significant attention is given to measuring geometric magnitude.

In recent time, in the methodic press, problems connected with the use of theoretic-multiple concepts for formulation of the basic ideas of algebra and geometry courses are discussed. Adherents of the theoretic-multiple approach point out how uniformity may be achieved in this way, leading to the content of the course being grouped around a small number of fundamental concepts, and also to the possibility of a stricter introduction of the concepts. Opponents of the theoretic-multiple approach note its abstractness, leading to formalism in teaching, and first and foremost towards the content aspect of the questions studied. In the process of discussion, the concept was crystallised of the inexpediency of using a theoretic-multiple approach in those situations when it leads to the formulation of a concept outside the sphere of its subsequent application. The account of this debate shows the strength of demands for more application in the course and for raising the level of practical skills, defining the direction for the improvement of the textbooks for the algebra and geometry courses.

The traditional division of the mathematics course into algebra and geometry is something which is not welcomed by all methodic experts and mathematicians. Many are adherents of the concept of a unified course of mathematics. In general the idea that it is possible to find some more proven alternation of algebraic and geometric material instead of the given timetable of lessons is in itself rather banal. The concept of a unified mathematics course lays claim, however, to greater, even to an original, unification of algebraic and geometric content in the framework of a single course of mathematics. Reference to the gradual tendency to decrease the number of hours given to mathematics teaching in school is quite a strong argument for the adherents of a unified course. Apart from dependence on the direct results of this work, which still has to be justified by experiments, it must be admitted that the very posing of the question of the optimum structure of the work in algebra and geometry is extremely crucial and deserves every attention.

In general methodic problems of teaching mathematics in secondary school are of the greatest interest. Here there is still a vast field of unrealised possibilities, a part of which we have referred to above. It is therefore far from accidental that the greater part of research in recent years has been in this field. It is worth awaiting new discoveries here, the adoption of new approaches, the appearance of promising textbooks.

Dr Firsov commented on some of the characteristics of the mathematics course at this stage. It follows the natural order of mathematical development as, for example, in the teaching of fractions. There is a division from the 6th grade into algebra and geometry. The unification of mathematics is not necessarily desirable - are we trying to create a false unity by 'linking the unlinkable,' or are there subdivisions of mathematics, and if so, how many?

The course does not include probability and statistics because this could not be developed far enough to be significant. Mathematical skills are not developed at any stage unless they can be applied at the same stage. If developed without applications, the children forget them.

The existing texts have been heavily criticised and a second wave of mathematical reform has begun. The geometry texts created the most problems. They were based on geometrical transformations which gave rise to difficulties in proof from Grade 6 onwards. The course could not be entirely deductive but depended on a small system of axioms and statements to be accepted without proof. Very precise language had to be used which made for clumsy definition. Some (eg congruence, vectors) were especially criticised. Also there was not a good stock of exercises.

Before turning to the new experimental texts, Dr Firsov asked why we teach geometry and showed the fallacy of some of the traditional answers. A systematic course is not needed to allow a 'visualisation' of the world. Drawing and measurement can be taught without geometry. Logical thought can be developed through language equally well. True that geometry fulfils all these aims, but do they all have to be taught in a simple course? Dr Firsov gave his own justification for the place of geometry in schools - that it is a unique scientific exercise, which leads from a simple beginning, with quick development, to significant conclusions.

Since 1979, a new series of texts by Pogorelov has been tested and is likely to be introduced in Russia and the Ukraine in 1982, and later in the Republics. It is based on a more traditional approach, using the congruence of triangles, but all the facts are proved. The idea of 'proof' is introduced in simple ways at 6th grade; more formal proofs of theorems come later.

The books have been designed to enable children to work independently. Their success has been compared with other texts, judged by teacher choice, and by the level of achievement of the children, and is shown to make geometry easier to understand.

It was agreed in the ensuing discussion that items used in the USSR for testing children's understanding of these texts might be exchanged with items used in the UK by APU.

7a SCIENTIFIC AND METHODOLOGICAL RESEARCH INTO PROBLEMS CONCERNING THE IMPROVEMENT OF MATHEMATICAL EDUCATION IN THE USSR

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The methodology of mathematics teaching is developing in the closest relationship with many branches of science. Its methodological basis is dialectical and historical materialism. It is an organic part of the general theory of teaching, didactics. On the basis of the principles of teaching and education established in didactics, the methodology of mathematics teaching determines the goals of mathematics training. The development of instruction methods and of matters concerning the organisation of teachers' and pupils' time is impossible without utilising the achievements of psychology and the physiology of the higher nervous system. Furthermore, the methodology of mathematics teaching makes use of the findings of natural, technological and other sciences in working out questions concerning polytechnic teaching and strengthening practical trends in mathematics teaching. There is a perfectly evident line between the methodology of mathematics teaching and the science of mathematics, which represents the basis of the content of mathematics as a teaching subject. Comparison of the results achieved by the methodology of teaching mathematics on the one hand and other subjects on the other is mutually enriching for all these sciences.

The research methods employed in the methodology of mathematics teaching are basically the same as those employed in didactics. Chief among them are the methods of historical and theoretical analysis, observation, experiment, the study of documents reflecting the activities of teachers and pupils (lesson plans, summaries and timetables, written and oral responses by pupils, etc).

The methodology of mathematics teaching is at present an independent scientific discipline and is one of the branches of pedagogics. The subject of research into the methodology of mathematics teaching has been defined, as has its place within the system of the other sciences, and a start has been made on developing research methods, while some serious problems on which the whole organisation of education in mathematics depends have been raised and partially solved.

The solution of problems facing the methodology of mathematics teaching is being pursued in the USSR Academy of Pedagogical Sciences (by Full and Corresponding Members, by the Mathematics Teaching Laboratory of the Institute for Scientific Research on Content and Methods of Instruction, and by the Mathematics Laboratory in the Scientific Research Institute SHOTSO (=School Organisation and Teaching Aids?), in teacher-training institutes in Union Republics, and in pedagogical institutes of further teacher training. A large role in developing the methodology of mathematics teaching is played by leading mathematics teachers. Degrees and titles of Candidate and Doctor of Pedagogical Sciences in the methodology of mathematics teaching are awarded for research and results achieved in this field.

During this period of scientific and technological revolution the methodology of mathematics teaching has once again had to face with particular seriousness the issue that there is a disparity between the content of the mathematical course and the demands the course has to meet. V L Goncharov is working on the problem of considerably strengthening its functionality, A I Fetisov on the need to give pupils a knowledge of the idea of geometrical transformations, and A I Markushevich is researching into the possibility of enhancing the role of mathematics in general education. A N Kolmogorov is engaged in the theoretical and practical aspects of introducing the idea of geometrical transformations into the geometry course. In the development of the methodology of mathematics teaching his work 'The scientific bases of the school mathematics course', which offers a solution for many methodological problems and outlines the directions in which the mathematics course should be restructured, has a particularly noteworthy role.

One of the achievements of methodological science is the concept advanced by A A Lyapunov of conducting ontodidactic research with a view to improving the structure and content of textbooks. A A Lyapunov means by ontodidactics a branch of pedagogical science which, for pedagogical purposes, processes or works on the contents of what is taught in one science or another and in so doing uses the methods of that science itself. A major ontodidactic objective was formulated in 1962 by A I Kolmogorov, who called on mathematics methodologists and pedagogues:

a to put the local foundations of the school mathematics course into a state in which it would be accessible to children aged between 12 and 15;

b to eliminate the discrepancy between the rigorous methods used in mathematics and non-rigorous reasoning devices used in the natural sciences and natural-science-related subjects.

During the 1960s virtually all scientific/methodological research was aimed at providing the textbooks and teaching aids required as Soviet schools made the transfer to mathematical education with a new content.

In connection with the move to universal secondary education, the system of ten years schooling was retained.

On the basis of research by Soviet pedagogues and psychologists it was considered desirable to limit the primary phase in Soviet schools to three years, thus attaching Class 4 to the secondary phase and also including that year of teaching in the system of teaching by subject. This was achieved by reorganising the traditional teaching material and dropping certain facets which had lost their importance (standardised problems, concrete numbers, etc).

The content of the initial mathematics course proved able to be enriched as far as conceptual thinking was concerned by tapping unused reserves of comprehension in junior schoolchildren. It became possible to acquaint pupils in the secondary and senior school phases with a number of topics of great importance in present-day science, technology and culture.

The selection of material to go into the school mathematics course is as a rule determined by the need to investigate in secondary schools those topics which are of broad general-educational value, which promote the formation of a dialectical-materialist outlook, which give the clearest and truest idea of the historical process of the development of mathematics and of the essence of modern mathematics, of the role it plays within the system of the other sciences and in technology and the national economy, and also other topics which would provide the optimum training of secondary school pupils for practical work in any sphere of the national economy or for continuing their education in a higher educational establishment. This requirement is met, for example, by including the elements of mathematical analysis in the curriculum, by introducing the idea of vectors into the school course, as well as some knowledge of the working principles of computers, and by substantially changing the role and content of problems and exercises. The curriculum now reflects the desire deliberately to develop mathematical thinking, with particular concentration on developing pupils' logical thinking and giving them a precise understanding of the abstract nature of mathematics. The new mathematics curriculum also reflects, if not overtly, the idea of mathematical structure.

A considerable amount of work has been done by scholars and methodological specialists in translating into reality in the new mathematics textbooks the results of much research. The excellent 'Mathematics' textbooks for Classes 4 and 5 were produced by a group of authors led by A I Markushevich. These textbooks contain not only arithmetical but also algebraic and geometrical information; they also contain the simpler theoretical and multiple concepts. They are in fact an organic combination of a textbook and a book of problems.

The new mathematics textbooks for Classes 4 and 5 fulfil highly complex functions as regards mathematics teaching. Their contents reflect many links between mathematics and the world around us.

In order to assist teachers, the scholars and methodologists have produced a number of manuals which, taken altogether, form the necessary instructional and methodological aids for teaching mathematics in Classes 4 and 5. They include a textbook, a teacher's manual relating directly to that textbook, class-teaching material, technical teaching aids (tables, models, calculating instruments, etc), material on the scientific foundations of the teaching material for the classes concerned, a manual for out-of-class work and a handbook of articles 'On teaching mathematics in Classes 4 and 5' in the 'Mathematics teachers' library' series, containing information on a selection of the most rational teaching methods and techniques.

The textbook 'Algebra' for Classes 6-7 was written by a group of authors and edited by A I Markushevich. In compiling this textbook the opportunity was taken to strengthen considerably the functionality of the algebra course; material was carefully selected on algebraic manipulation so as to lead adequately into the further study of mathematics and closely allied subjects. Particular attention was given to working up the set of problems designed to inculcate concepts, and also to developing abilities and skills. The textbook 'Algebra 6-8' also forms part of the single complex of instructional and methodological aids for mathematics teaching.

The textbook 'Geometry' for Classes 6-8 was compiled with the assistance and under the guidance of A N Kolmogorov. It is a new geometry course compiled on the basis of the ideas of geometrical transformations and with a theoretical-multiple approach, making widespread use of vectors and vector algebra. It is also accompanied by a set of instructional and methodological manuals.

The textbook 'Geometry' for Classes 6-10 by A V Pogorelov is an axiomatic course containing relatively simple axioms. It is a textbook set out for the pupil like a conspectus, in summary form, and in this it differs from other current textbooks by virtue of its conciseness and brevity. It does however contain a set of problems covering study of the entire range of theoretical material. It is a textbook of a new type.

The experimental textbook 'Geometry' for Classes 6-9, compiled with the assistance and under the guidance of V G Vostyansky, is interesting in its underlying concept. It strengthens the logic of the geometry course and makes widespread use of mathematical symbols. The textbook defines many types of learning activity engaged in by pupils. It contains a wide selection of problems including some with a practical content. Both of the two last-mentioned textbooks are at present at the stage of experimental testing.

The textbook 'Algebra and the fundamentals of analysis' for Classes 9-10 was compiled by a group of scholars led by A N Kolmogorov.

The effectiveness of the instructional process, during which a person's mental and moral self are formed, depends in large measure on successful mastery of an educational content identical and compulsory for all members of society, and on the comprehensive satisfaction and development of the spiritual needs, interests and abilities of each individual schoolchild. In order to offer pupils the possibility, within secondary schools, of deepening their knowledge in the physico-mathematical and natural sciences and humanities, and in order to develop pupils' varied interests and abilities, a new form of instructional work was introduced in the USSR with effect from 1966 - optional studies according to the pupil's choice. The inclusion of optional studies in the teaching curriculum is a further development of the fundamental ideas and principles of Soviet general-educational schools. While not violating the principle of a unitary schooling system, the introduction of subjects for study by choice does help to raise the level of general education.

The general-educational functions of optional studies consist in offering opportunities to students who show an interest in or an aptitude for a subject to acquire additional knowledge, abilities and skills in that subject.

We can therefore draw an important conclusion. Optional studies are linked with the basic course and have ground in common with various types of extramural work by virtue of the programme or optional studies on which both are based. Moreover, the programme of optional studies is flexible, making it possible to vary the content of the optional course in directions which correspond with the possibilities and wishes of teachers and pupils in optional studies groups.

Optional studies have played a positive role as a widely-based experimental testing-ground for introducing into schools new features in the basic mathematics course (testing the new content, developing and establishing instructional methods for these new features). One could quote quite a list of new topics in the school mathematics course which have been 'run in' and had their content and associated instructional methods polished entirely through optional studies.

Optional studies have arrived in the Soviet general-educational schools as a new way of organising for the differentiation and individualisation of teaching mathematics to pupils. It should be particularly emphasised that optional studies have given the teacher and the devisers of teaching methods a broad range of options as regards using methods and forms of individualisation which, for one reason or another, have proved difficult to use in mathematics lessons.

Optional studies have made it possible to carry out the deliberate and systematic selection and further development of gifted schoolchildren. During their initial years, optional studies largely copied the forms of selection and development of gifted pupils which were used in schools and classes where individual subjects were studied in particular depth. Later, optional studies began to throw up their own specific forms and methods of working.

Optional studies have coped and contined to cope successfully with the task of giving pupils a professional bias. This was particularly noted when introducing such special optional courses as 'computer mathematics', 'optimisation methods' and 'programming'. Not only do these options provide a pretty detailed and scientifically well-grounded knowledge of numerical methods and of the pecularities of research into mathematical models of important present-day mathematical applications linked with the use of computers, but they also train pupils to acquire the kind of up-to-date applied mathematical 'baggage' which a young person can utilise later in his practical work. Statistics show that such optional studies do quite consistently help students to develop a stable interest in applications of mathematics, and this does much to determine the choice of direction in which their continued education will proceed.

One of the results of research done by methodological science is the development of a new form of enhanced mathematical training for students, ie the secondary general-educational schools with in-depth study of mathematics and its applications. The founding father of such schools is S I Shvartsburd. Under his guidance and with his participation a curriculum has been developed, together with the contents of mathematics instruction in such schools, and associated textbooks have been written. The system of enhanced mathematics training for students that has been developed within secondary schools has become the exemplar for the development of corresponding forms of teaching in physics, chemistry, biology and other subjects. Since their inception, such schools have produced a large number of young people with a very good training in the field of mathematics and programming. Such schools can exist only in large towns or cities, where there are highly-qualified mathematics teachers and major enterprises possessing computer centres.

In order to offer schoolchildren living far away from large towns the opportunity to acquire, while still at school, an enhanced mathematical training, we have organised in our country the so-called All-Union Mathematics Correspondence School, attached to Moscow University. Scholars have produced a curriculum, established the course content, and written textbooks and methodological manuals for this School. The School works with schoolchildren from places remote from Moscow who show a heightened interest in mathematics. To work with the schoolchildren it employs a large number of teachers from higher educational establishments, helps the children to acquire a truly sound mathematical training, and purposefully prepares students for making a conscious choice of profession. This school has brought to light many able students in mathematics. Similar mathematics schools have now been set up attached to many other of the country's higher educational establishments.

One of the links in the unitary system of enhanced mathematics training for students in our country is the organisation which works on the popular science and physico-mathematics journal 'Kvant', whose chief editors are Academicians A N Kolmogorov and I Kikoin. The journal, which is designed for schoolchildren who show interest in mathematics and physics, enables them to enter the world of mathematics and mathematics books, and to envisage a possible future specialising in mathematics. It opens up possibilities for undertaking one's first independent mathematical project, and is open for publishing contributions from schoolchildren and teachers. Much that is published in the journal is original.

Thus the adoption by Soviet schools of education with a new content, and the move bringing the school mathematics course closer to the ideas, concepts and methods of present-day science, are profoundly progressive developments which have given a powerful stimulus to scientific-methodological research in the field of mathematics teaching methods.

Editor's note

The Soviet participants submitted an additional paper, reproduced hereafter, giving in more detail an example of the kind of curriculum research described in the previous paper. This paper is based on the doctoral thesis of the author (who was not present at the seminar), and concerns research directed towards the development of effective textbook exercises to teach skills of problem-solving, and to use problem-solving to strengthen pupils' mathematical understanding.

7b TEACHING MATHEMATICS THROUGH PROBLEM-SOLVING, AND TEACHING THE SOLVING OF PROBLEMS IN THE SECONDARY SCHOOL

Dr Y M Koliagin (USSR)

The effective organisation and management of teaching pupils to solve different mathematical problems is a most important way of developing a high mathematical culture in pupils, and of making the teaching of mathematics more active. It is precisely in solving mathematical problems that pupils master consciously and firmly the system of mathematical knowledge, ability and skill. More than this, in the process of solving mathematical problems, pupils, in the most natural way, can form qualities of personal creativity which are necessary for them to take an active part in the creation of material and spiritual values in the future, regardless of the profession they will choose.

The scientific and technical revolution has in many ways necessitated the incorporation into the school mathematics course of new material. However the change in the content of school mathematics basically implied only a change in the mathematical content of educational tasks; their inner structure, the technology of their organisation in education, the function realised by them is not unified in practice, it has not undergone the radical changes necessary in this plan. As before, the selection of mathematical problems for the teaching of many topics in the mathematics course frequently has a subjective character. Some problems in the school mathematics course are either too trivial or too complex; in the exposition of certain topics of the mathematics course, the leading ideas underlying the content and methodics of the solving of problems etc were not adequately reflected.

In the contemporary methodics of teaching mathematics there is still a strong tendency to reduce problems to a subsidiary role, to explain, to help master and consolidate teaching of pupils' theoretical material or to form particular mathematical skills in them. Contemporary practice of teaching problem-solving with wider didactic aims is weak.

Meanwhile, as formerly, in contemporary mathematics teaching a large part of the teaching time is taken up with the solution of problems during lessons, and in homework. The ineffectiveness of this use of class time tells negatively on the quality of mathematics teaching as a whole, and on the quality of the preparation of pupils for practical work in conditions of contemporary production.

In connection with this, it is appropriate to remember the saying of the well known pedagogue-mathematician G Polya ('Mathematical discovery', 1970):

> 'What does mastery of mathematics mean? It is the ability to solve problems, not just the standard ones, but also those requiring a known independence of thought, common sense, originality, inventiveness.
>
> 'Therefore the first and chief duty of a mathematics course for secondary schools is to underline the methodic side of the process of solving problems.'

Regardless of methodic usages and advantages, the solution of problems is always a key stimulus to the awakening in pupils of an interest in the study of mathematics. The results of a questionnaire we took of pupils in Classes 6-8 graphically showed this. Of four motives listed in the questionnaire for creating an interest in the subject - the contents of the mathematics course, the practical significance of mathematics, the skilled teaching of this subject by the teacher, the solution of problems - the last occupied the leading place.

True, it must be taken into consideration that an interest in mathematics through the solving of problems comes about when the idea of the problem set is understood by pupils, when the problem set interests them by its content, form or method of solution, when during the process of solving the problem they have the possibility of thinking for themselves, of showing quick-wittedness, initiative, creative elements. These conditions usually arise in solving problems which are non-standard for pupils, problems for which the method of solution is unknown to them.

However the matter is not so much in the standardisation of the problems themselves in school mathematics, as in the standardisation of the methodics of their presentation to the pupils. It is characteristic, for instance, to have a collective solution in class, for pupils to work after a model, involving only a very little independent work.

But to change in the necessary direction of destandardisation is not easy. The content of the problem material is closely linked with the content of the theoretic material being studied; you cannot substitute the problems of a mathematics course with the problems of 'quickwittedness' which do not depend on specific mathematical content. Destandardising the methodics of problem-solving, we inevitably come to the need more frequently to use a problem form of teaching. Strengthening the role of independent work by the pupils in the solution of non-standard problems, we inevitably come to the need for individualising teaching. Both things demand a significant amount of teaching time which the teacher, as a rule, does not have.

A serious contradiction arises, which could be solved only by way of an essential restructuring of the methodics of using problems in the teaching of mathematics.

A list of the specific questions of the methodics of problem-solving in the teaching of mathematics, answers to which are of great practical significance, is sufficiently extensive. Thus, in the future, we must define exactly the function of problems in the teaching of mathematics, to discover a system of problems which realise the idea of teaching mathematics to develop and morally educate, to define the quantitative and qualitative minimum of problems which are necessary in order to realise through them this or that concrete aim in the teaching of mathematics. Working out a concrete methodic of teaching pupils by solving mathematical problems, it is worth establishing the basic mental skills which can and must be formed in the pupils in the process of solving problems, to identify common methods of solving problems with which it is possible and useful to acquaint pupils, to work out in the right way the methodics of teaching pupils by the application to problem-solving of mathematical methods (eg the method of comparison, the vector and co-ordinate methods, the method of geometric transformations), to define the parameters of the system of problems which control the degree to which pupils have been taught and are being taught and their mathematical development at each stage of mathematics through problems, etc.

In Soviet pedagogic science many of the methodic problems formulated above have been solved, while others are undergoing intensive research.

The working out of issues concerning the application of problems in school mathematics teaching shows its two basic aspects : the teaching of mathematics in the process of solving problems, and the teaching of pupils the solution of problems by common methods.

The first aspect leads, in the main, to search for ways of activating the teaching material, to securing the mastery of the curriculum material; the second, to the optimisation of the mental development of pupils, to the formation in them of creative skills.

The unity of these two components in the teaching process, their effective functioning optimises the methodics of developing mathematics teaching, in which problems are leading didactic means.

One of the most useful ways of optimising the methodics of teaching mathematics in the process of solving problems, and teaching pupils the common methods of solving problems, is the construction of theoretical models, regulating the formulation of problems and the process of solving them by pupils.

A systematic psychological approach to the building of theoretical models in the application of problems in teaching mathematics showed the expediency of the following position.

The problem is the special condition of the system (S, R), where S is the subject (the person resolving the problem), and R is some system, composed of real (or abstract) objects connected with a definite sum total of attributes and relations. In this complex system (S, R) it is possible to separate two basic conditions by the relations between its components S and R: the standard situation, and the problem.

The demand indicated above is often expressed in the form of a specific special indication, connected with plural R and pointed at the same time to the problematical nature of system R and to the desirability (or necessity) of its elimination (for instance, to solve an equation). The solution of problems is considered by us as a method of transforming the given problem situation into a standard one. The process of solving problems is seen as an activity of a person in the direction of searching for solutions.

We will consider now the model we have constructed of the educational problem (we note that we call every problem used in school mathematics teaching an educational problem; we note likewise, that we understand the term 'problem' in a broad sense: our own problems, exercises, a theorem for formulation or proof, and so on).

In the structure of an educational mathematical problem it is possible to separate four basic components: the condition of the problem, the aim of the problem, the solution of the problem, the basis of the solution. A short designation is UORZ. In the terms of this model all problems the solution of which is found by mathematical means we call 'mathematical problems'. If all four components named above are present in the mathematical objects, the problem is called 'purely mathematical'; if only O and R - the mathematical subjects - the problem is called an 'applied mathematical problem'.

The model considered here of the understanding of the educational mathematical problem is further developed in the model which reflects the typological pecularities of the problems depending on the number of separate basic components, which are unknown to the solver (or badly defined).

School mathematical problems are either problems with deliberately known methods of solution, or problems for which the method of solution is practically defined by the theoretic subject matter being taught at the time. In this way, for many mathematical school problems, in practice all the basic components are given; they are offered as training exercise, directed towards the formation of some mathematical techniques on the basis of skills already formed. Meanwhile, as research has shown, the formation of thinking in pupils is closely linked with the ability to solve problems with two or three unknown components. We note that in psychology and cybernetics analogous problems are characterised as problems of a creative character. Problems with one unknown (or poorly defined) component we will call 'teaching' (we remember that such problems are the most characteristic in school teaching), with two - 'search', with three - 'problems' (problems in a non-mathematical sense). In the framework of the given typology problems called 'standard' find their own place.

Research has shown that the model of the typological pecularities of school problems described above can at the same time function as a model of the process of solving problems. Precisely in the process of solving problems, made into the form of a problem (not with the mathematical sense), the problem is successively transformed into a search, a teaching and a standard problem.

In stating a search problem, the number of transformations is naturally reduced. We remember that in teaching we have as a rule to deal with the last or penultimate step of this transformation (with standard or teaching problems), which, as research shows, significantly lowers the didactic value of the teaching problem.

Research has shown that the given model can be accepted in the capacity of the structural basis of a methodic system of school mathematical problems. This is conditional first of all on the fact that the relationship of the educational problem to one or other type is dictated by the methodics of reasoning. It depends less on the concrete content of the problem than on the form of its posing, or on the individual pecularities of knowledge and experience of the pupil to whom this problem is set (or by the expected knowledge and experience at a given moment of the group of pupils in a particular class).

In this way there is the possibility of systematically composing problems of the new types on the basis of almost any problem in the school mathematics course, by modifying the conditions of the problem (the diversification of the level of the problem of each of the four isolated basic components) or of the time of its presentation to pupils. This opens up the possibility of using a limited set of school problems (and even one and the same problem) in different didactic aspects; many standard problems can become non-standard problems in specific teaching situations. It is important to note that the methodics of constructing, in conformity with the given model, in practice allows one simply to carry over an ordinary problem into the category of problems of a creative character. The didactic significance of the problem grows sharply because of this, and the extra time needed is extremely small. In the same way the methodics are realised in practice in the mass teaching of mathematics.

The effectiveness of the use of problems in teaching mathematics to a great extent is defined by the formation in the pupil of a definite combination of thinking skills which we call the ability to solve problems. In this complicated complex there is, besides definite thinking skills, also the ability actively to apply the mathematical knowledge and experience which they have. 'The ability to solve problems' on one hand is formed in pupils in the process of solving problems, and on the other hand itself serves as the basis for the mastering by pupils of the standard ways of solving problems, being its concrete manifestation.

Research shows that the formation of one or another concrete thinking skill, contributing to 'the ability to solve problems', happens unequally; the extent of its appearance depends to a considerable degree on which stage of the process of solving the problem its subject has reached, and also to which type the problem which is being solved belongs in the given concrete conditions. These thinking skills function more effectively when the problems being solved by the pupils are those we call 'search' and 'problem' (in the non-mathematical sense). They are developed more readily in the first and last stages described above. However it is precisely these conditions which are often not observed in teaching mathematics. Therefore an effective methodic for developing the teaching of mathematics using problem-solving must include the clear arrangement of these components.

In particular, when teaching pupils the solving of problems, it is worth giving special attention to teaching the first stage of the process, to the teaching of the search for a solution, to systematising mathematical knowledge and experience in the process of solving problems. It is worth using in teaching corresponding systems of heuristics. These heuristics (partially presented in the works of G Polya) can be constructed on the basis of known methods of teaching knowledge, and also on particular thinking skills. Our searches show that the timely teaching to pupils of the use of a combination of these or other heuristics ensures not only a more successful solution of this or that problem, but more importantly helps the pupil to master standard ways of solving a problem, and at the same time fosters a purposeful development of important capacities in their thinking.

In this way, at any particular stage of teaching mathematics, problems are a means of education, without application of which an active and solid mastery of the curriculum by the pupil, the pupil's many-sided moral upbringing and development, and the introduction of a creative character into his work, is impossible.

In the long term plan for possible further changes in the content of the school mathematics course in the near and visible future, it is worth particularly underlining that it demands a simultaneous and adequate change of the whole system of school mathematical problems.

It is possible to foresee that the teaching of mathematics in secondary schools of the future will, in essence, set itself the process of solving a definite methodically thought out succession of school mathematical problems, in the course of which the mastery of the bases of mathematical knowledge, as also of the moral education of pupils consistent with the demands on time made by mathematical and general culture, will be ensured.

The Institute for Scientific Research into Content and Methods of Instruction is the body responsible for organising research in mathematics education. Every five years reports on existing research projects are called in. The Institute then co-ordinates and directs the future programme of research and selects leaders, both for their qualifications and for the quality of their previous work. Further degrees may be awarded. The results are put into practice in mathematics teaching. The focus of research during the past ten years has been on syllabus content; research efforts are now shifting more towards methods.

Some of the current questions being researched are:

- the best way to lay out a textbook,
- the criteria and methods for assessing texts and pupil attainment,
- the ideal construction of a set of exercises so that concepts are formed,
- the use of applications in mathematics teaching,
- the mathematical training of the ablest students in Grades 9 and 10.

Professor Monakhov described some of the testing already done. When the new syllabuses were introduced in 1970 in country areas, the same tests were given to 7th grade urban pupils working on the old syllabus and to the 6th grade rural pupils who were working on the new syllabus. The rural pupils' scores were between 70% and 80% while the older urban pupils' scores were around 65%. Normally, urban pupils' scores are higher than rural pupils' of the same grade. In another investigation, in mathematics, physics and chemistry, pupils in urban and rural schools, and Russian and non-Russian schools, and pupils of teachers with more or less experience and higher or lower qualifications, were tested. In Professor Monakhov's opinion, this kind of assessment should be repeated every year and the information processed.

In answer to questions, Professor Monakhov said the better results of the rural pupils could have been due to improvements in the text (transformations were introduced at a more appropriate time and the exercises were more carefully sequenced). He agreed that lack of success may be due either to the introduction of an inappropriate topic, or to bad presentation, but to discover which experimentally would take several years, and there is not time. He discussed the use of exercises in more detail, first to establish a concept, then to apply it, and then to make constant use of it.

On the question of the failure to attract girls to mathematics, he said that in the USSR there was no difference between the sexes in attitudes to, or achievement in, mathematics. On the amount of the syllabus actually taught, he thought that the workload was too heavy even though the syllabus and texts have obligatory sections and optional ones.

Editor's note

The British participants were particularly interested to note that Soviet research was described almost entirely in terms of curriculum work. There was no mention of psychological studies, in striking contrast to mathematics education research in Britain.

7c RESEARCH IN MATHEMATICS EDUCATION IN THE UK

Dr A J Bishop, Department of Education, Cambridge University

The context of research

As a result of having an educational system which places much emphasis on local control, rather than on centralised control, many decisions and choices are made locally about mathematical content and teaching procedures. Increasingly those choices are being influenced by research, whether carried out locally or nationally, and the ideas of 'research and enquiry' influence most features of mathematics teaching in the UK today. Experimentation is also built into our system at all levels and teacher training courses in the UK emphasise strongly the skills of innovation, individual teacher problem-solving, and decision-making.

It is therefore rather difficult to describe coherently what is happening in research in mathematics education in the UK. However for the purposes of this paper and the discussion which will follow I propose to omit further reference to innovation and trial-error experimentation, and also not to describe local action-research and decision-orientated inquiry. What I will focus on is research which uses systematic, controlled and well-documented enquiry procedures in order to increase generally our understanding of mathematics teaching. But in considering such research, its trends and its values, the reader must always remember the context of the system, and the general usage of enquiry, research and experimentation throughout it.

Research of the type to be described is carried out partly by the staff of the National Foundation of Educational Research, funded by the Government, by Local Education Authorities and by educational institutions. It is also carried out by staff at universities and colleges, either funded by government agencies such as the Social Science Research Council, or by industry through agencies like the Nuffield Foundation. It can be done by individuals working in any branch of the educational system, and by research students studying for higher degrees. Very, very few people work as full-time permanent researchers, so nearly all of this research is done either by temporary research staff or by people for whom research is only part of their work responsibility. Dissemination and discussion of research ideas takes place through books and journals, at conferences organised by teachers' organisations such as the Association of Teachers of Mathematics and the Mathematical Association, and through the various activities of more research-oriented groups such as the British Society for the Psychology of Learning Mathematics.

The content of research

Turning now to the content of research, I will focus on the significant developments which have taken place during the last ten years. For research trends prior to that, the reader is referred to Bishop (1972).

There is still much interest on the cognitive aspects of mathematical learning and thinking. Skemp's work (1979) on schemas and understanding continues to develop, and his research with Mellin-Olsen, from Norway, on the differences between relational and instrumental understanding (Skemp, 1976) has been a stimulus to many people. Errors and misunderstandings are still a fruitful source of research data and Rees (1973) and the CSMS

(Concepts in Secondary Mathematics and Science project, based at Chelsea College, University of London) team (1981) have shown how productive their study can be. The CSMS work in particular has shown how systematic mathematical development is throughout secondary school, and has produced much information about relative difficulties and stages in learning important mathematical ideas.

Processes and abilities have offered another set of constructs with which to interpret children's mathematical behaviour, and Bell (1979) discusses process research generally and indicates well the potential of these ideas for improving teaching. Burton (1980) on problem-solving, and Booth (1981) on children's own methods for doing routine computations, also show this potential. Much of this research is now carried out in a clinical mode, and is beginning to offer a different perspective on research on abilities, which traditionally (in our country) typically used the psychometric methods so well criticised by Krutetskii (1976). For example, spatial abilities are of great interest to mathematics educators (Bishop, 1980), and the ideas of processes and processing, coming from individual studies, are beginning to have a marked effect on our ideas about spatial abilities. In particular, visual processing is now coming under close scrutiny (Bishop, 1980). Work by Tall and Vinner (1981) on intuition also relates to these ideas, and shows particularly the power of intuition in controlling mathematical thinking.

Language and mathematics interact in many ways through teaching, learning, reading from textbooks, and using symbolism in solving problems, and Austin and Howson (1979) in their excellent survey of the research literature show the importance of this area. Of growing interest are the problems children experience when reading mathematics (Rothery, 1980) and the difficulties immigrant children have learning mathematics through a second language (Dawe, 1981).

Another developing research field concerns the affective aspects of mathematics learning. Traditional attitude research told us rather little about actual feelings towards mathematics, but recently Backhouse (1980) has been analysing what it is that children find of interest in the subject. Perhaps the more dramatic impact however has been created by Hoyles (1975) and Buxton (1981) with their work on anxiety towards mathematics. When one reads accounts of the fear generated in mathematics classrooms one is forced to wonder why this aspect has never really been researched to the extent others have. It is surely a very important area.

Moving on to research on teaching it is interesting to note that since my previous review there has been very little development in research on teaching methods, but much more focus on what actually happens in the classroom. Morgan's (1977) perceptive study shows well the problems faced by teachers when they try to change their methods from class teaching to an individualised system. Yates (1978) documents clearly the power wielded by the individual teacher in the classroom, and when in addition one reads Nash's (1973) work one can see the effect which teachers, perhaps unknowingly, can have on pupils' ideas about their own abilities. Girls seem to be particularly affected by this and there is a great deal of interest generally in the problems which girls seem to have with mathematics (Preece, 1979).

The language of classroom interaction also offers very revealing data and one project concerned with the analysis of this is the Classroom Interaction Project (1978). As with any developing research area, the methods of enquiry are also having to be developed for classroom research and as well as interaction recordings, the critical incident technique (Bishop, 1976) is being developed and is also producing some interesting data. As methodology improves we will be likely to see more research taking place within existing classrooms, as well as a change of focus - away from merely collecting data and towards more analytic and theoretically oriented research.

Conclusion

Finally I must point out that I have not attempted to cover exhaustively all the recent research in the UK on mathematics education. Space has forced me to focus on what I see as the significant developments in research, and to give references which are examples of those developments. (Editor's note: More detailed documentation of research was available at the seminar.)

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Dr Bishop emphasised that the system, being decentralised, demands informal decision-makers at all levels. Those who make use of the findings of research are curriculum developers and textbook writers, classroom teachers, head teachers, and heads of the mathematics departments, producers of audio-visual aids and educational tests, and the large group of teacher trainers. Recent research has, for example, contributed to the production of the Mathematics Teacher Education Project materials.

The benefits of research lie not only in the findings but in the formulation of methods, in the materials (tests and apparatus) which are produced, and in the data (test-scores, children's errors, transcripts and video-recordings of the lessons) which can be used by other workers. Some research consists of surveys of what is happening while other is of experiments to find out why it is happening. Reports give an account of the methods used for each, and are freely available to everyone who may be interested.

12a THE IMPROVEMENT OF THE QUALIFICATIONS OF MATHEMATICS TEACHERS IN THOSE SOVIET SCHOOLS PROVIDING A GENERAL EDUCATION

N A Yermolaeva, Deputy Director, Department of Schools, Ministry of Education of the USSR

The contemporary scientific and technical revolution and the transition to universal secondary education in the USSR raise the role and significance of the improvement of the qualifications of teachers to a new level.

In the Soviet school questions concerning the improvement in the qualifications of teachers were always considered, and are now considered, as questions of moment for the whole state. To a great extent the quality of teaching and moral upbringing of the rising generation depends on this. Therefore the improvement of knowledge and the raising of the qualifications of teachers are regarded not as a personal concern of this or that teacher, but as a state concern as one of the most important civic duties of the teacher.

In the USSR a harmonious system of improving the level of qualification of teachers was developed long ago. Its basic contents are as follows:

- the development of knowledge of the system of Marxism-Leninism,
- the acquisition of new knowledge, reflecting the achievements in contemporary scientific and technical thought and production in the fields connected with the discipline being taught,
- the improvement of the general pedagogic and systematic qualifications of the teachers

The most important role in the business of improving the scientific qualifications and systematic ability of those teachers who already have the appropriate education is played by advanced training institutes. In this respect continual work is done by pedagogic committees in each school, and by methodic unions of teachers by subject (school, group and region).

There are advanced training institutes (IUU) for teachers in each oblast, territory and autonomous republics, and in each region methodic laboratories have been set up under the direction of the advanced training institutes for teachers and the local state organs which have responsibility for the peoples' education.

In the work pattern of the advanced training institutes, teachers go approximately once in five years to courses and seminars. They are released from their professional obligations, but retain their earnings. There are likewise theoretical and practical conferences, 'pedagogic readings' and so on. In the general system for improving qualifications a significant place is occupied by various kinds of conference, among which periodically convened regional conferences of teachers of the same subject have an especially important place. Their practical significance is that side by side with the discussion of a ser- - of theoretical questions, illuminated by new achievements in corresponding branches of knowledge, they facilitate the wide dissemination of the professional experience of the best teachers on the basis of comprehensive discussion at conferences.

The effectiveness of different forms of improving qualifications is measured in the first place by the extent to which they are combined with the systematic independent work of the teacher, with his self-education. The self-educative work of the teacher is closely connected with the teaching and guiding aspects of his work, with his preparation for work in school, and is considered in the USSR as an effective form of improving qualifications and of general development.

Self education among teachers has, in recent years, become very widespread. Recommended programmes of self-education for teachers of every speciality are centrally worked out, themes and bibiographies are provided for the self-education of teachers in their academic subjects, in pedagogics and methodics of teaching, in questions of the theory and practice of educating their pupils.

A special place in the general plan for improving the qualifications of teachers is occupied by work for the improvement of the level of teaching skill. This work alerts teachers to possible mistakes, and helps them to examine particular difficult questions both on the content of the teaching material and in the methods of teaching it. The regional (town) pedagogic laboratory heads and directs the methodic work in each region. In the school itself the pedagogic committee plays a major role in defining the concrete content, the direction of work for the improvement of qualifications in pedagogic skill. Methodic work in the school and in the region is not only closely interwoven with the whole system of improvement of qualifications of teachers, but is an integral part of it.

The raising of the level of the academic preparation of teachers, and the contemporary demands of the content of general education, arise from the aims of the social and economic development of Soviet society and the contemporary scientific and technical revolution, and ever-growing demands are made on the system of improving the qualifications of teaching personnel. The solution of these problems can be found by the broad participation in the conduct of this work of study by academic research institutes and universities. So more and more academics, professional teaching staff from the higher pedagogic institutes and universities, are drawn to work on courses for the improvement of the qualifications of teachers. In recent years universities and pedagogic institutes have organised courses for the improvement of teachers' qualifications, using their own academic premises and laboratories.

The modernisation of the content of education, a transition to new academic plans and programmes carried out between 1965-75, has had an enormous importance for secondary schools providing a general education. Particularly significant changes were made in the curriculum for natural sciences and mathematics, particularly mathematics. The new curriculum is based on principles of teaching on a high (but in accordance with the capacity of the pupils) level of difficulty, on a quickened pace for mastering the academic material, and on the raising of the amount of theoretical knowledge. It is necessary to note that the improvement in the content of the curriculum was not just a case of adding new themes and sections to traditional courses. The main feature in the improvement of the new curriculum was the changing of the structures of subjects taught, the raising of their academic, theoretical level in conformity with contemporary academic views.

The modernisation of the content of education demanded likewise the improvement of teaching methods, the wide use of such modes of teaching as maximise the thinking process, train in systematic, independent work, nurture the capacity to compare, to analyse, to make deductions.

The considerable changes in the content of school mathematical education and in teaching methods demanded a change in the manner of improving the qualifications of mathematics teachers. A decision was taken that all mathematics teachers should have an annual retraining course.

The training of teachers to use the new curriculum was done not only through the courses run by advanced training institutes, but also by the whole system of methodic help to the teacher in situ, which in recent years has become significantly more active. Ways of doing this have become more diverse, while the content has been enriched.

Much attention is given to work with methodic specialists. Special courses are run for directors and teachers in key schools, for tutors at base schools, for directors of method units, and for schools of advanced experience.

All teachers are included in collective forms of methodical work in their schools. In the organisation of work with pedagogic personnel, departments of the people's education and their methodic laboratories were run on the rule that only those teachers could be allowed to work with the new curricula who had passed through the retraining course. Teachers who, for one reason or another, had not passed through the retraining course in time, were involved in the work of the permanent special seminars at the regional methodic laboratory which the personnel administrators organised for them, using experienced teachers who had passed through the course. The courses in the advanced training institutes took three to six of the best teachers in every region (town) and trained them as lecturers. They conducted the seminars, which took place in the regions (towns) on days when there was no school (once or twice a quarter). At each session some complex topic, to be studied in the following month, was considered.

A sharp increase in the number of courses demanded a large number of lecturers and instructors for practical work. Therefore republic (central) advanced training institutes gave special attention to the preparation at special courses of groups of lecturers for oblast, inter-region and urban IUU. The methodologists in IUU and in departments of the people's education, the teachers in pedagogic institutes, the best teachers who had, as a rule, the experience of excelling in work with the new curriculum, went through such additional training. Advanced training institutes regularly, four to five times a year, conducted instructional, methodic conference-seminars for the heads of regional methodic laboratories on questions of the procedures for selection and sending them on courses, the conducting of regional seminars, the planning and organisation of methodic work and the self-education of teachers and directors of schools. Courses for the directors of schools are differentiated (they are nominated either as inadequately prepared, or as experienced); in this their pedagogic specialisation is taken into account. These courses make it possible to resolve such problems as the retraining of directors of schools, and also the improvement of their academic, theoretical and methodic knowledge in their subject. With the aim of giving help to the oblast IUU in the conducting of courses and in making uniform demands on teachers, the Ministries of Education of Union Republics organise the preparation of a body of lecturers and consultants for regional and town departments of the people's education. The work of courses is organised on the basis of the IUU, and also on the basis of pedagogic institutes, teacher training colleges, state universities, and departments of mathematics in higher technical institutes. These educational establishments provided the material basis for courses and allocated the best teachers for the carrying out of this enterprise. With the object of giving help to the teacher in mastering the new content of mathematics education, a centralised provision for teachers was organised with an educational component in which, besides pedagogic methodic oversight, there is a set of didactic materials for each class (task cards for independent pupil work, texts for control work, material in the form of problems and exercises for final revision and preparation for examinations, and for differentiation of work with pupils). These training aids served the teacher as a reference point in defining the level of demand for knowledge, for skill and work habits in pupils, they contained concrete methodic instructions for each topic and additional information to the textbook, and they explained the fundamental concepts of the author in the choice of material and its structure. The publication of a series of 8-10 books ('Library of mathematics for the teacher') was organised. (Editor's note: this works like a book club, with teachers receiving one book per month on advance subscription). Among these publications are books on the methodics of the teaching of the subject, books which describe the fundamental trends in contemporary science, the history of its development, recommendations for equipment, for the furnishing of a class room and the use of technical aids in the teaching process, for out-of-class activities with pupils and for optional studies. Among the books in this series is a textbook for implementing links between subjects in teaching. There is a reference book in the series of mathematical terms and concepts.

The further broadening and improvement of all existing forms and methods for improving the qualifications of teaching personnel will promote the improvement of the training of the specialists of a complex and responsible profession - of the teachers and directors of the people's educational establishments.

Mrs Yermolaeva presented the advanced training institutes as the link between teachers in the classroom and researchers in education. The heads of the institutes are brought together each year so that new developments in education can be passed on to them for dissemination among the teachers. Ideas originating in individual classrooms may be shared with institute staff who may then spread them more widely.

Every five years teachers have to be re-certificated. A Commission inspects the teacher, and makes training recommendations which are made both to the teacher and to the institute. Teachers then undergo further training, either full or part-time. The majority attend the institute full-time for between two and four weeks in the holidays. If performance doesn't improve they can be demoted to lower classes, or given other work outside the classroom. In recent years, all mathematics teachers have undergone annual retraining, in view of the particular demands placed on them by changing syllabuses and textbooks.

12b SUPPORT FOR THE SERVING MATHEMATICS TEACHER

B J Wilson, Head of Consultancies Group, The British Council

The British educational system places much responsibility on the individual teacher, not just for teaching effectively according to a predetermined programme, but for making decisions as to the nature of that programme. It is a system that assumes a high level of professionalism in teachers, an assumption whose justification in practice varies widely from school to school, and from individual to individual within the teaching profession.

An essential requirement of such a system is that professional advice and support should be freely available to all practising teachers. Such support is seen as the key to the steady improvement of the quality of British education. It is more important than revised syllabuses, better textbooks, reorganised schools or new teaching equipment.

Professional support for the serving mathematics teacher takes a wide variety of forms, and is available through a wide range of channels and organisations. This paper attempts to list, in note form, these channels. It must be emphasised, however, that all this provision is entirely voluntary. A teacher who wants help, information or advice has a wealth of it available to him, and many teachers make excellent use of such support. Too many, however, ignore it, and there is no way in our system that such teachers can be compelled to benefit by any form of in-service training or assistance. Professional responsibility and individual freedom are the two most noteworthy characteristics of the role of the British teacher.

Within the school

Other mathematics teachers
Head of the Mathematics Department (particularly at secondary level)
In-school in-service courses

Through the Local Education Authority (LEA)

Mathematics Adviser (whose full-time work is to provide information and advice to mathematics teachers)
Teachers' Centres (more used by primary than by secondary teachers)
Local in-service courses
Working groups of teachers from a group of schools. 'Primary maths guidelines' have been produced by such working groups of teachers, usually with the assistance of the Mathematics Adviser, in many LEAs

National provision

Her Majesty's Inspectors (HMI)
  - visits to schools
  - publications and reports

Department of Education and Science (DES) reports, eg the Cockcroft Committee, curriculum bulletins
Schools Council - publications, projects

Through colleges and universities

Short courses

In-service diploma courses

In-service degree courses: BEd, BPhil, MEd, MA, MSc, etc; some full-time, some part-time, some mixed

Advice from some sources, eg Shell Centres for Mathematical Education (at Universities of London and Nottingham), Centre for Studies in Science Education, Leeds

Open University - courses, publications, TV programmes

Through the professional associations

The Mathematics Association (MA)
- Diploma in Mathematical Education
- Diploma for Teachers of Low Attainers
- annual conference
- specialist conferences
- local branches
- occasional publications and reports
- 'Mathematics in school', 'Mathematical gazette', mini-magazine for teachers of 4-11 year olds

The Association of Teachers of Mathematics (ATM)
- 'Mathematics teaching'
- annual conference
- local branches
- occasional publications and reports

The Institute of Mathematics and its Applications (IMA)
- publications, day-conferences

Teachers' associations

Occasional publications of eg the Assistant Masters and Mistresses Association (AMMA)

Through projects

Project materials

In-service courses

Occasional conferences

BBC

Schools radio, television

ITV

One school television series

Publishers and equipment manufacturers

Visits to schools

Inspection copy service

General publications, eg the 'Times educational supplement' has occasional maths supplements

8a DETERMINATION OF THE MATHEMATICS REQUIRED IN EMPLOYMENT BY YOUNG PEOPLE BETWEEN 16 AND 18 YEARS OF AGE.

A Fitzgerald, Faculty of Education, University of Birmingham

Introduction

The intention here is to describe one of the projects set up at the request of the Committee of Inquiry into the Teaching of Mathematics in Schools (the Cockcroft Committee) in order to provide data for that committee to consider in the fulfilment of its brief. This particular project was entitled 'Mathematics in employment (16-18)' and was under the direction of Professor D E Bailey of the University of Bath. The writer of this present paper was seconded to the project from the University of Birmingham as a research fellow. In addition there were two research associates and two consultants, one from industry and another from the University of Bath. A Steering Committee was also set up, consisting of representatives from the Cockcroft Committee together with the project team.

Aims

The project set out to obtain a picture of the mathematics used in their employment by young employees between the ages of 16 and 18 years. It was requested by the Cockcroft Committee that in the time available for the field work (about ten months) as wide a range as possible of industrial situations and occupations should be considered. Geographically the sample was to be drawn from various areas of England and Wales and was to include small, medium and large companies.

Preliminary work

Relevant evidence submitted to the Cockcroft Committee was made available to the project. This was read and the principal points were noted.

In recent years prior to this project many small studies and projects related to the problem had been conducted by individuals or small groups from education and industry, often with support from Local Education Authorities. Professor Bailey had earlier compiled a catalogue of these activities which varied widely in their aims and methods. ('A survey of mathematics projects involving education and employment', D E Bailey, University of Bath.) Any papers that had been produced were obtained and studied. They included reports of discussions between teachers and employers, results of a scrutiny of mathematics tests used in the processes for selecting suitable young employees, collections of examples from work situations illustrating some uses of mathematics, and results of surveys usually involving the use of questionnaires. There were also available a few dissertations by individual students reading for higher degrees. These studies vary considerably in quality and most have limitations of one kind or another, but many of them have some positive contribution to make to understanding of the field.

Methodology

Many of the prior studies were based on discussion about the mathematics involved in employment rather than direct observation of it. As a consequence Bailey and Fitzgerald both believed that there was a need for more first-hand study in detail of the mathematics used in employment. It was agreed that the principal thrust of the project's work should be to examine the mathematics young people were using and how they were handling it, by directly observing and discussing these issues with them in their work situations. In addition it was considered desirable to study the broader context within which each employee was functioning and the relationship of this context to the employee's use of mathematics.

Highly structured observation and interview techniques were rejected as tools for the research for various reasons and instead researchers operated within an agreed broad structure. This allowed them the freedom to pursue any line of investigation which looked promising and to adapt rapidly to any shift of emphasis requested by the Steering Committee.

Details

The following extract from the letter sent to companies which members of the research team planned to visit illustrates in more detail how data was to be collected:

> 'It is intended that the researcher should:
>
> (i) observe young people in training and on the job, and discuss examples of their work with them and with their supervisors in order to determine the mathematics actually involved;
>
> THIS IS SEEN AS THE FUNDAMENTAL ACTIVITY OF THE STUDY
>
> (ii) cover the whole range of ability of young employees from those with no paper qualifications through to O levels;
> (iii) study the range of occupations on both the technical and commercial sides of employment, including areas where mathematics may not be very apparent as well as those where its use is explicit;
> (iv) compiled from the study a collection of examples of applications of mathematics;
> (v) relate the findings to procedures for selecting recruits, in particular any tests of a mathematical nature;
> (vi) relate the findings to employees' attitudes towards and performance in mathematics at school;
> (vii) relate the findings to further education courses which employees are following;
> (viii) identify later demands that may be made on an employee as a result of a first promotion;
> (ix) consider possible changes in mathematical requirements which may occur in the future on account of advances in technology and new business methods.'

It may be helpful to enlarge on some of these points. Numbering of the comments that follow corresponds to that in the above extract.

(i) Once the field work was underway, the value of looking at the actual work the young employees were engaged in and discussing it with them soon emerged. It was found that young people often seemed unaware of mathematical aspects of their work and would sometimes say initially that little or no mathematics was involved in it. In some cases this was true, but in others the actual tasks they performed would reveal some features of a mathematical nature, albeit often very elementary. The point is illustrated by the comment of one young lady, who 'hated' mathematics at school and obtained no qualifications in the subject. Her work involved converting kilograms to pounds using a calculator and rounding the answer to the nearest pound. The interviewer said, 'This looks like mathematics to me,' to which the employee replied, 'This isn't mathematics: it's easy!'

It was also found that supervisors of young employees were not always aware of the details of the mathematics their charges were using nor of the methods they adopted, illustrating another advantage of getting as close to the work as possible.

Training schemes posed problems for the researchers, since it was soon found that in some situations such schemes were broadly based covering a range of skills a young trainee could, but not necessarily would, be called upon to use after training. Where possible, and if time allowed, observation and discussion of the work of trained personnel was also included as a source of comparison with training schemes and to enable researchers to make some judgement of the extent to which mathematical features met in training needed to be mastered.

(ii) It needs to be borne in mind that many but by no means all of the more able pupils (as judged by success in the GCE O level examination) would go on to full-time education beyond the age of 16 years and so would not come within the project's study.

(iii) Certain fields of occupation were excluded from this project's work because they were being covered by another project based at Nottingham. Excluded areas included agriculture, medical services and occupations in financial professions such as banking and insurance.

(iv) At present the examples of applications of mathematics found during the project are still embedded in the project's reports and do not form a separate collection.

(v) Different companies use tests of widely differing natures. Some of the tests have been produced by professional agencies, others have been compiled by personnel in the companies. Wherever possible tests were discussed and analysed to determine their relevance to the work that recruits would be called upon to undertake.

(vi) Initially it was the attitudes of young employees that were considered, but as the project progressed attitudes of some more adult supervisors were also explored. Interviewees were asked what their feelings were towards mathematics when they were at school and also what reasons they thought there might be for every pupil having to study mathematics at school up to the age of 16 years.

(vii) Many young employees are granted release from work for one day per week, or for a block of several weeks, to follow a course of study at a college of further education. Although the mathematics studied at college was not strictly within the project's brief it was thought necessary to give some attention to it and how it relates to employment, as part of the overall context. It may be considered that it is more appropriate for some aspects of mathematics required by young employees to be studied in further education courses rather than at school.

(viii) It was thought to be important not only to look at mathematical demands that would be made on young employees soon after leaving school, but also how those demands might change with subsequent promotion, because if employees were not following any further education course they could still be depending on ideas and skills they had learned at school. 'Promotion' was loosely interpreted to include any work on to which an employee might be moved after training or as a result of accumulated experience, even if the move did not necessarily carry any higher status. For this part of the project's study it was, of course, necessary to observe and discuss the work of older employees. There are also some occupations, such as within the chemical and foundry industries, where by law employees must be over the age of 18 years. Recruits to these occupations could also be depending on aspects of mathematics studied at school and so were included in the investigation.

(ix) Attempting to anticipate what future demands are likely to be was perhaps the most difficult part of the project's work. As the sample built up visits included a variety of situations involving modern technology and related production and office methods. On the assumption that the newer systems will become more widespread the changes effected by them on the mathematics that young employees required were seen as possible indicators of future trends.

Selection of the sample

The possibility of constructing a sample of occupations directly was considered but rejected on account of a number of problems that this would have involved. Instead it was decided to build up a sample of companies which provide some representation across the diversity of industry. There is a document published by Her Majesty's Stationery Office called 'The standard industrial classification' which delineates various categories and sub-categories of industry. This was used as a guide, with particular categories being allocated to each researcher, whose responsibility it was to locate and obtain access to relevant companies. In this way it became possible to investigate a wide range of occupations peculiar to particular industries such as glass, china, leather, chemical, printing, etc, as well as occupations such as engineering crafts and clerical functions which are found across a range of different industries.

Reporting visits to companies

A report of each visit was written up following an agreed broad format. In particular two acronyms were invented in relation to the fundamental activity of the study, namely:

> STIM, which stood for 'Specific Tasks Incorporating Mathematics'; under this heading reports described tasks in which young employees were engaged and within which mathematics was embedded;

MIST, meaning 'Mathematics Incorporated in Specific Tasks'; tasks were analysed both together with the employees and afterwards by the researcher, endeavouring to identify the precise nature of the mathematics involved and the methods adopted by the employee in handling it.

All reports were submitted back to the companies visited for corrections of error in fact and for additional comment. They were also circulated among the research team and then despatched to London to be available for reading by members of the Cockcroft Committee.

Reporting visits to the Steering Committee

Regular meetings of the Steering Committee were held to which written summaries of progress and verbal reports were submitted. In return the Committee offered advice and comment on the methods adopted, possible shifts of emphasis and so on.

Reporting visits to the Cockcroft Committee

At the request of the Steering Committee two interim reports were produced for the Cockcroft Committee summarising progress and containing provisional findings. A final report was submitted in August 1980 in which some earlier impressions were reinforced and others modified. After introductory chapters this report consisted of two broad sections. The first described various categories of occupations that had been investigated, the mathematics that had been found within them and the context, as outlined earlier in this paper. The second part consisted of a series of papers by the researchers on particular aspects of the investigation which they deemed to be of importance. This material, which is likely to be of considerable interest and significance to all those concerned with mathematics education, will be available after the publication of the report of the Cockcroft Committee itself.

In his presentation, Mr Fitzgerald gave an account of his experience, working on the project 'Mathematics in employment (16-18)' introduced in his paper.

The young people interviewed were mostly of the lower ability range employed in jobs such as unloading machines, assembling components, and glassblowing and cutting, or as laboratory technicians or in different clerical occupations. The total list of mathematical items discovered in use ranged from elementary arithmetic to the trigonometry of the right-angled triangle. Most would be found in any examination syllabus and at first sight there is a close correspondence between the needs of employment and the school mathematics courses. Closer investigation proved this to be untrue.

Many employees need no mathematics, eg assembly line workers. Of the remainder, each individual requires only a few mathematical items, and these are usually simple arithmetical procedures. Only a few need more complex skills. In most circumstances, the complexity of the task is reduced to eliminate errors, and the need for mathematical skills may be removed entirely by the use of calculators, automatic counters, dials marked with safe limits, fixed distance calibration or computer control.

It was found that school methods were rarely used in employment. Multiplication and division were done with a calculator or by methods peculiar to the task. Mathematics at school is often introduced in abstract terms while at work it is learned through practical situations. Employees reach a high level of competence through frequent use. In particular they are in a good position to assess the accuracy of their calculations, and unlikely to disregard the size of error which results when a calculator is wrongly used.

They are often required to carry out self-checking procedures (eg cross-casting) or to check one another's results. The mathematics taught in school is often much more comprehensive than is needed; for example, a wages clerk 'needs to multiply fractions', but the fractions involved proved to be halves and quarters and could be turned to decimals and the calculator used. In the building industry measurements were made in mm to avoid the use of the decimal point.

It was rare to find a worker using algebraic fractions, and never quadratic or simultaneous equations, or algebraic graphs. The only skill required was to substitute numbers in formulae. If a transposed form of the formula was required, it was provided.

In engineering and building construction industries a knowledge of elementary geometrical facts concerning angles, circles and parallel lines was needed. When geometrical reasoning was required, a formula was given.

The conclusion to be drawn from this investigation is that young people use very little of the mathematics learned at school in employment.

Another project looked at the mathematical needs of everyday life. Few of the people who were asked why they thought so much mathematics was taught when it was not used again by so many could give a convincing reason, although they felt there must be one.

The resulting discussion on the value of learning mathematics was continued later in the meeting.

8b THE STUDY OF MODERN APPLICATIONS OF MATHEMATICS IN SOVIET SECONDARY SCHOOLS

Prof V M Monakhov, Deputy Director, Institute for Scientific Research into Content and Methods of Instruction, Academy of Pedagogical Sciences of the USSR

The reform of mathematics education that took place in the 1970s in those Soviet schools which provide a general education considerably broadened the spectrum of what is studied by senior pupils in the contemporary applications of mathematics. This is the more significant as the USSR moved into universal compulsory secondary education.

In this paper attention is mainly concentrated on the selection of new content which reveals contemporary applications, on the search for pedagogic ways of introducing it into the school course of mathematics, on the evaluation of the significance of the new applied subject matter in the field of teaching and upbringing in the total development and education of the young person in a socialist society in schools which provide a general education and in polytechnics.

In considering the wider use of contemporary applications of mathematics in school education, it is worth remembering the extension of the boundaries of mathematics in contemporary science and the global use of computing in the use of mathematical methods both in traditional and in newly arising fields of application. Another aspect of the contemporary scientific and technical revolution that it is worth considering is the use of scientific methods which make possible the optimum planning of the economy through progressive organisation of social production. It is precisely these objective tendencies which have appeared in the improvement of the school mathematics course and in the increased use of applications.

An interesting aspect of the first tendency is the growth of the role of digital mathematics in its various applications. We note that the term 'digital' reflects in essence the digital character of computing. Many 'digital' or 'finite' parts of mathematics have become, in recent time, extremely important for their applications; for instance, mathematical logic acquired a decisive importance in the development of the theory of computer programming.

The second tendency was conditioned solely by the great importance which the so-called 'optimisation tasks' have acquired in applied mathematics. The search for solutions to practical problems led to the development of new applications of mathematics: optimum control, operational research, linear and non-linear programming, the theory of games, the theory of encoding information, stochastic programming.

These two tendencies in the development of contemporary mathematics have led to the introduction into the curriculum of school education work linked with contemporary applications of mathematics, and involving mathematical processes which are fully accessible to senior classes. Work of which the mathematical, polytechnical and educational value evoked no doubt included:

a computing mathematics and programming, which allows pupils to become acquainted with the digital character of computing and the encoding of information, by the language of communication between man and computer, by the particular uses of numerical methods in resolving every possible problem by computer, b methods of optimisation, which allow pupils at many stages to become acquainted with important extremal problems and specific uses of mathematical methods (in particular, methods of linear programming) for the discovery of the optimal solution of economic problems.

The introduction of new subject matter connected with contemporary applications of mathematics in the teaching process in school has necessitated the use of new methods and new didactic means. In particular, in an obvious form in schools, new kinds of mathematical models have appeared, algorithms for solving a problem and their programming, the interpretation of the mathematical solutions obtained, and so on. Educationalist-mathematicians have been faced with new problems of the organic inclusion of new methods of teaching applied aspects of mathematics into a traditional school methodic system, which has already been in existence for many decades.

It is interesting to note that this organic inclusion came about comparatively quickly. This was facilitated by the fact that the introduction of contemporary applications in the subject matter of school education was brought to pass by an extremely rational scheme: at first in an experimental way new subject matter was approved in schools with a deepening of the study of mathematics and applied mathematics; if the results of this teaching confirmed the fitness of the principle of the new subject matter for school practice, then this new subject matter served as the future basis for the development of teaching material for the optional mathematical studies in schools of general education. The second stage of the experiment already encompassed a significantly wider contingent of pupils. The whole complex of methodic problems in the introduction of one or another topic into the course of mathematics for the schools of general education was decided precisely here.

Before going on to a description of the results of the introduction into the school of contemporary applied mathematics, one would wish to draw a conclusion about the psycho-pedagogic character of the process of introducing applied mathematics into school. Firstly, the best conditions for raising the level of mathematical education can be created only when the school pupils have the possibility for concrete application and direct practical use of this higher knowledge. Extensive experience of experimental teaching of the applications of mathematics shows that it strengthens the use of the pupils' mathematical knowledge, and this is resulting in an increase of applied mathematics in the school course.

Secondly, the development for schools with a mathematical specialisation, and optional studies with special courses, of work in programming, computer mathematics and methods of optimisation showed that the raising of the level of mathematical education is directly connected with vocational teaching. It is precisely the element of vocational teaching in programming skills which really makes serious demands on the general mathematical training of pupils and gives scope for the direct application of this improved preparation. This dual link between mathematics and programming contains within itself methodic possibilities still far from fully exploited for the further improvement of secondary mathematics education.

Thirdly, the introduction of applied mathematical knowledge, researched over a number of years, is distinguished by a breadth of practical applications of mathematics in contemporary production. This leads to the development, to a significant degree, of a new and more economic manner of thinking, founded on the broadening of mathematical understanding, and of the use of mathematical modelling and algorithms, and of improved economic competence.

Fourthly, the whole process of working out and introducing into school contemporary applications of mathematics has one difference in principle: the methodics of teaching were worked out and perfected at the same time as was the content of the course. This gave great possibilities for the effective inculcation of applications into the system of secondary mathematics education, and in fact turned the schools with mathematical specialisation and optional studies into a huge experimental training ground for the working through of new subject matter.

In the 'Algebra' course in Class 9 (age 15-16 years) a section was developed 'Algorithms and elements of programming'. Here the pupils' comprehension of algorithms was crystallised and systematised. Languages are introduced to describe the algorithm: the language of the block diagram, and the programming language. The last serves as a form of communication between man and computer. It is worth noticing that the block diagram language introduced into school practice is a powerful didactic descriptive means by which many as yet unused possibilities for perfecting the traditional methods of teaching school mathematics are accommodated. It is also proposed to widen the pupils' comprehension of the system of numeration, that is, as well as the decimal system of numeration, to give a concept of the binary system, providing the arithmetical basis for the functioning of computers and other cybernetic equipment. In the course 'Algebra and the rudiments of analysis' in Class 10 (age 16-17 years) the applied orientation of the work on equations and inequalities is significantly strengthened with the inclusion of linear programming problems. It is important to note that the appearance in a plain form in the subject matter of general secondary mathematics education of contemporary methods of optimisation (methods of linear programming) is an exceptionally progressive methodic achievement. This is not just the solution of individual examiners of minimum and maximum problems, but the acquaintance with contemporary numerical methods of optimisation achieved by a great algorithmic community.

Now in school a successful way of developing the ability to solve optimisation problems has been formed: at first, questions are looked at as being at the extremum of a function of one variable, and optimisation as a geometrical problem, then problems accessible to linear programming are resolved.

In this way the programme and subject matter of school education contains sufficient material for the formation of applied knowledge in the pupils, and for skill in the operational use of mathematical methods in different contemporary uses.

Prof Monakhov compared the UK approach in which the needs of employment were investigated to see how the syllabus should be changed, with the approach of the USSR in which the syllabus was devised to meet the future need of a 'computerate' society. The course was first introduced in the specialised mathematics schools twenty years ago, then in the options available in general secondary schools, and finally in the courses available for all pupils. At first it contained the algorithms of programming and elements of linear programming, but the latter proved to be less important and was not introduced into all schools.

Over the twenty years, there have been three generations of computers, programs have developed and computer languages have changed. School mathematics cannot keep up with the changes but must use the elements of programming in mathematical education as a whole. This is seen as part of a child's general education and it broadens the sphere of applications of mathematics. Optimisation especially is a skill likely to be needed more and more in the future.

Prof Monakhov could see the place of an investigation such as that described by Mr Fitzgerald in devising exercises for use in the vocational schools of the USSR.

9a THE CURRENT STATE OF DEVELOPMENT OF THE MATHEMATICS COURSE FOR SENIOR CLASSES (AGES 15-17) IN SOVIET SECONDARY SCHOOLS, AND FUTURE PROSPECTS

Dr V V Firsov, Head of the Mathematics Teaching Laboratory, Institute for Scientific Research into Content and Methods of Instruction, Academy of Pedagogical Sciences of the USSR

The move to universal compulsory secondary education in the USSR coincided in time with the introduction of the most radical reform of secondary school mathematics education that has been seen during the years of Soviet power. A major element in this reform was the realisation of one of the long-standing dreams of the country's progressive teachers - the introduction into schools of the elements of differential and integral calculus.

Since the times of the All-Russian Congresses of Mathematics Teachers at the beginning of the 20th century the idea of introducing the elements of calculus into schools had been a constant feature in the works of virtually all the major pedagogical scholars in the field of mathematics teaching methodology. Even earlier, far-sighted words had been said on the subject by the great Russian mathematician P L Chebyshev.

The leading Russian pedagogues clearly understood that the study of calculus would give the school mathematics course an integrity, a completeness which it had hitherto lacked. The traditional course represented, in effect, an agglomeration of special devices aimed at solving narrow classroom problems. This inevitably led to the use of stereotyped problems and examples solvable by means of these special devices. A pressing need was felt to introduce into the course some powerful mathematical methods making possible the solution of broad categories of major practical problems. Perhaps the most significant of such methods is that of the calculus, and the introduction into schools of the rudiments of mathematical analysis has powerfully enhanced the potential of school mathematics in both its polytechnic and applied aspects.

Consequently the conviction has grown among mathematicians and pedagogues that acquaintance with the elements of differential and integral calculus must be a compulsory component in the system of general education. In this sense the introduction of so-called 'higher mathematics' into schools has not subverted the teaching aim of providing a general education, nor has it been contradictory to the changed status of secondary education, which has now become compulsory in our country.

As is well known, the system of education of senior schoolchildren in the USSR is a differentiated one. Those who complete eight-year schools may continue their education in the 9th and 10th classes of secondary schools, or they may enter technical/vocational training schools or secondary specialised educational establishments. Nevertheless it is obligatory for each educational establishment to require that the mathematical training of pupils completing its course be at a particular level, as laid down in state curricula. At the present time the mathematics courses in the senior classes of schools and in technical/vocational schools virtually coincide, whereas the ground covered in the mathematics course in the majority of secondary specialised establishments is much greater than that in the schools. And so, having first described the mathematics course for Classes 9 and 10 in secondary schools, we shall offer a fairly specific criticism of the system of mathematics training for senior schoolchildren in the USSR in the system's major component.

Mathematics teaching in Classes 9 and 10 in secondary schools is carried out within the framework of two independent courses: 'Algebra and the rudiments of analysis' and 'Geometry'. These courses are respectively a continuation of the two separate mathematics courses for Classes 6 to 8.

As is evident from its title, the algebra and rudiments of analysis course - apart from the study of the elements of differential and integral calculus- also provides for the study of certain aspects of algebra, though these do not form the basic content of the course. They include firstly some aspects of trigonometry and also a large section devoted to the solution of various algebraic and transcendental equations and inequalities, and systems of equations.

In current textbooks the exposition of the elements of calculus is preceded by a systematisation of the information acquired by pupils in eight-year schools about real numbers. Differential calculus is then further set out in the completely traditional manner: the boundary constant, a constancy of function, the derivative and its application. Naturally, many propositions have to be accepted without proof, as special axioms. The exposition of this range of topics is completed by acquainting pupils with the simplest differential equations.

A small amount of integral calculus is studied in Class 10. The purpose of this is to introduce into the school mathematics course methods for measuring geometrical magnitudes; previously, measurement of the magnitudes of the more complex spatial objects was done by semi-primitive methods.

It is easy to see that the rudiments of analysis course, being constructed in the way described, is a copy, adopted for schools, of the traditional course in the higher (further) educational system. As it would have appeared, it might have been considered right to rely on the considerable experience of the upper schools during the preliminary stage of introducing calculus into schools.

But it is becoming daily more and more clear that there is a pressing need to develop a fundamentally different teaching methodology designed for schools. Among the problems that arise here we consider the most important to include consideration of the possibilities for setting out calculus on the 'physical' level of rigour, putting emphasis on pupils' physical and geometrical intuition. Also of interest is the possibility of narrowing the range of functions investigated by analytical methods, and this will probably maintain a quite high level of rigour and lose little as regards the possibility of solving applied problems. And finally, several exploratory pieces of research demonstrate the need and the feasibility of beginning studying calculus in Class 8, ie one year earlier than at present.

The results of these researches will undoubtedly have an impact on the teaching of calculus in schools. At the same time, it would be wrong to wait until this research is completed and to abandon making improvements in the existing course. Among the basic lines of improvement in the near future we would single out the need to overcome excessive formalisation during the initial teaching stage. With this in mind attempts are at present being made to replace clumsy definitions and proofs in the language of $\epsilon - \delta$ by their non-formal analogues.

A rather peculiar position in the course is occupied by the fundamental question of how to complete the line of development of the concept of number that was started back in primary school. It is generally accepted that the natural culmination of this line in schools should be familiarity with complex numbers. The introduction of complex numbers completes not only the subject of number but directly generalises propositions advanced earlier about the solution of algebraic equations. During the implementation of the reform, because of shortage of teaching time, the study of complex numbers was dropped. At the present moment spare capacity has been found for re-establishing the subject in the course, as it is so important both generally and philosophically.

The solid geometry course in secondary schools is entirely traditional apart from questions concerning geometrical transformations of space and the use of the 'apparatus' of integral calculus for the measurement of geometrical magnitudes. The directions in which this will be improved in the near future largely depend on what system is adopted for the instructional exposition of plane geometry. Thus the main lines for improving the teaching of solid geometry coincide with the lines identified in the previous paper for improving the geometry course in the 6th to 8th Classes.

As we see, within the mathematics course for senior classes there is considerable potential for improvement still untapped, and this is a major and rewarding field for the researchers, the pedagogues in mathematics. The majority of these problems, which are apparent even from so fleeting an investigation of the courses, have only a slight connection with mathematical problems as such, though they are much more closely linked with pedagogical problems.

It is nevertheless quite possible that decisive help will come in fact from the mathematical side. The concept, advanced in the works of Corresponding Member of the USSR Academy of Sciences A A Lyapunov, of the ontodidactic processing of teaching material, ie its own restructuring being effected by the particular branch of science itself (in our case, mathematics) for teaching purposes, will be implemented, it is to be hoped, in research by professional mathematicians and will provide the methodology of mathematics teaching with solutions of many current problems which are not achievable by conventional methodological means.

In his presentation, Dr Firsov began with a challenge - why is it necessary to teach calculus as part of general education? Who needs it, apart from those going on to further education with a technical or mathematical bias who will learn it more quickly in higher education?

But, he claimed, mathematics is more than the sum of its different parts - there is interdependence between them. Calculus is the only example of a powerful method which broadens the scope of the applications of mathematics. When we introduce it we show how problems can be solved by mathematical methods, and we fulfil an educational, rather than a utilitarian, purpose. Further, it enables us to pose real-life problems, not just textbook ones.

As an example of the need for educational grounding he quoted the class of problem 'how can x be made most effective with least expense?' The turn of phrase is incorrect but people will cease to know that it is inaccurately expressed if they do not have sufficient educational background. It is our duty to transmit the mathematical ideas essential to this background in our teaching.

The Chairman remarked that we were delighted to have these views expressed, but that in the UK our problem was to reconcile utilitarian and educational aims in practice. Attention was drawn to the difference between objectives and actual achievement. Calculus is an important means of putting into mathematical terms many everyday experiences, but the full concept is difficult for every pupil to understand and many retain only a formula which doesn't help them to understand eg the annual rate of inflation. Probability and statistics is another topic in which understanding rather than skill is vital to everyday life. Examinations often require the use of skills rather than a general appreciation of the concept. Should essay-questions appear in mathematics exams, as they do in every other subject of the school curriculum? Only in mathematics do we expect the subject to be learned 'as a trade'. Some research has been done on the achievement of educational aims but not enough is known on how to measure the quantities desired.

In conclusion, Dr Firsov said that it was important, but difficult, for teachers to attend both to basic skills and to general appreciation of mathematical ideas, and the pendulum would swing from one to the other.

*Probability and statistics is another topic in which understanding rather than skill is vital to everyday life.

9b AGES 16-18: PREPARATION FOR ADVANCED STUDIES

J Hersee, Executive Director, The School Mathematics Project

This paper concentrates on those students who are preparing to study at university or polytechnic. As a basis for discussion, five main issues are raised, with comments on them from an English viewpoint. As with the earlier years, there is no national syllabus for mathematics. It is common for a student to continue his studies after school at a university or polytechnic in a different part of the country from his home.

Should all academic pupils study mathematics during these two years?

In England no subject is compulsory. The choice of subject to be studied is made by students with guidance from their teachers. English (literature) is the most popular subject. Mathematics is studied by about 12.3% of all students who continue their studies during these years after the end of compulsory schooling - by 16.6% of boys and 6.9% of girls.

Over the last twenty years the numbers of students studying mathematics during these two years has increased. An official report suggested that all students should study mathematics during these years. Should this be so? Can it be achieved?

What proportion of the student's time should be devoted to mathematics?

In England, students study only three academic subjects, each for about five hours, with four hours homework each week, together with general studies and a subsidiary subject, as shown in Figure 1. While the majority of those who study mathematics intend to become scientists, engineers or mathematicians, a significant proportion will go into medicine, geography, economics or social science. There is no national curriculum, but all students follow programmes of study of the pattern shown.

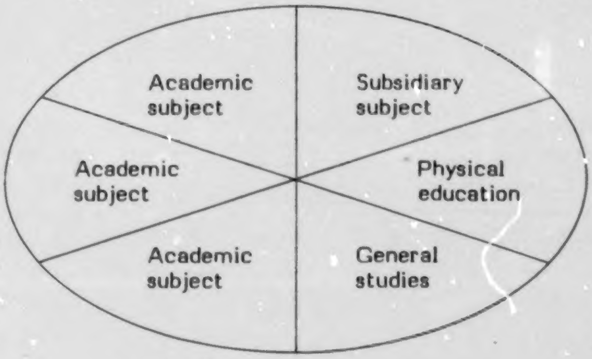

Figure 1

It is also possible for students to study mathematics as two of their three academic subjects; many of those who do so become engineers (see Figure 2).

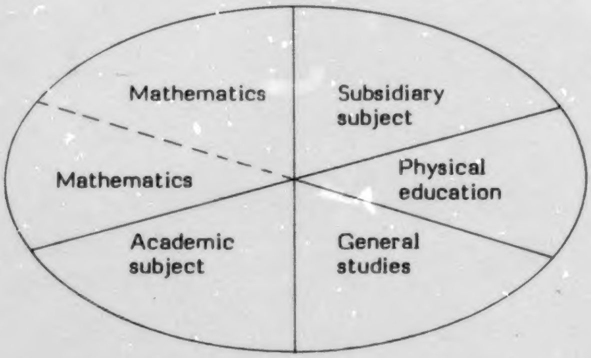

Figure 2

Would intending scientists, engineers, etc benefit from the study of a wider range of subjects, with less time given to mathematics?

If a higher proportion of all students is to study mathematics, how much time should be given to it?

The content of the syllabus

What should be included in the mathematics syllabus?

It is a long-standing tradition in England for syllabuses to include both pure mathematics (algebra, trigonometry, geometry, calculus) and applied mathematics. The latter was once exclusively Newtonian methanics, but the term now includes probability and statistics.

Although there is a considerable amount of mathematics common to the different English syllabuses, chiefly in pure mathematics, they differ greatly in their 'applied' content. A student may study only mechanics, or only probability and statistics, or something of both.

Should applications form part of the mathematics course, or should it include only pure mathematics?

Is it possible to include a reasonable amount of both mechanics and probability and statistics in the time available?

How should the syllabus be determined for a student whose later studies will be in an arts subject which requires no mathematics?

How should the syllabus be taught?

In England, a major influence on the teaching is the examination which students take at age 18. This is an 'external' examination, set by an examining board outside the school and not set by the student's teachers. Timed, written papers are used, but different styles of question may be set to test the same piece of content; the style of question influences the teaching approach.

Four examples of different teaching approaches follow.

(i) The traditional approach to the binomial theorem is by extension of the student's knowledge of elementary algebra, adding further brackets:

$(a + b)(a + b) \qquad (a + b)(a + b)(a + b) \qquad \text{etc,}$

followed by extended practice in the technique of working out expressions such as $(1 + 3x)^5$, $(1 - 2x)^7$, etc. Only at a much later stage is the result used; the tool is sharpened ready to be taken out of the tool-kit when it is needed.

A different approach would be via probability, where the binomial probability distribution would lead up to the binomial theorem via Pascal's triangle. The approach to the result and the practice of the technique would be in the context of application.

(ii) Work on simple harmonic motion could be implicit, but the mechanical significance not explained, in the solution of the differential equation

$$\frac{d^2y}{dx^2} = -k^2y\,.$$

The traditional Newtonian mechanics course would include standard problems dealing with the oscillations of a mass hanging from the end of an elastic string, but other courses would also use the theory as a simple model of, for example, what happens to the springs of a car at a depression in the road.

(iii) The beginnings of calculus would, traditionally, begin with consideration of the gradient of a chord of the curve $y = x^2$. Other cartesian curves would follow, leading to the standard rule for differentiation of x^n and a series of practice examples. The work would continue in a purely geometrical context for some time.

An alternative approach is to consider, for example, the motion of a ball falling freely under gravity. Measurements taken from a multi-flash photograph are used to calculate average speeds, leading to the idea of speed at an instant and then to the derivative. There is much numerical work and the kinematical context is emphasised and used to present the new ideas; only later would the geometrical developments, such as tangents and normals, be introduced.

(iv) In the teaching of applied mathematics the emphasis is often on the mastery of a number of principles and routine techniques. Students may achieve great facility in dealing with standard problems of types made familiar by repeated practice. There is little discussion of the assumptions made and the simplifications necessary to solve the problem.

In the selection of content and the way in which it is taught a balance must be found between developing students' interest and their technical facility, leading them to ask 'why?' as well as showing them 'how'. As a preparation for advanced studies both aspects are important.

What should the most able students do?

As stated earlier, it is possible for students in England to spend two-thirds of their academic study time on mathematics. The content of the syllabus they cover is about twice as large as the norm. They thus begin their advanced studies with a much wider knowledge of the subject. Is this the best way to help the able?

Would it be better for them to cover only the same content, but more deeply? In mathematics can we make a distinction between deeper study and adding more content to a syllabus?

Would it be better for able students to work more rapidly through the normal course and begin their advanced studies at a younger age?

Would it be better for them to use the time released by their ability to work faster for the study of other subjects?

The points raised above reflect some underlying concerns. The questions of syllabus content and teaching approach arise, in part, from recent attempts to present mathematics as a unified subject, with emphasis on its wide use in the study of other subjects and in the world today. This is linked with the aim of attracting more students, including a greater proportion of girls, to continue their study of the subject after age 16. Another factor to be borne in mind is the variation in motivation and ability in a class of students at this stage. While many of them will see the importance of mathematics for their future studies, others, who are told that they must study the subject to qualify for further education, will work with less dedication. Other members of the class may be uncertain of their future careers; the mathematics course must retain their interest and provide a general basis for their futures which will be in a variety of specialisms.

(**Editor's note:** Examples of GCE A level mathematics syllabuses and papers were made available at the seminar for reference, as were those for O level and CSE.)

Mr Hersee explained the terms 'applied', 'applications of', 'applicable', and 'applying' mathematics. Before 1960, 'applied' mathematics (mostly Newtonian mechanics) was taught with only a few links with pure mathematics. Mechanics was taught as an axiomatic study though the axioms were not clearly stated. In the early 1960s probability and statistics were introduced, but usually as a separate subject. The newer syllabuses include both Newtonian mechanics and statistics, but are overloaded. It has led to a view of pure mathematics which has emphasised its usefulness, and is described as 'applicable'.

'Applying' mathematics refers to the process of mathematical modelling, using material from any part of mathematics. It has been argued that students should see how mathematics is used, rather than simply acquire techniques. A further idea is that mathematics should be discovered in a problem or situation, rather than a formula be derived and then applied.

Asked how early he would like experimental teaching to begin, Mr Hersee said 'at five'. He thought it preferable to giving a set of rules to start with experiment and build up the mathematics on which it was based. This could then be used in other applications. He did not think it mattered if the less precise methods of science were used in the primary classroom because children receive experience as a whole, not in separate compartments. At secondary level the most important thing for most pupils is to use mathematics in the whole of their experience, and only the most able should move on to a systematic study after 16+.

The question was again raised as to whether the time spent in heuristic teaching justifies the result. Terms such as 'better long-term results' were balanced by 'a drop in attainment'. It was suggested that 'discovery' is in itself a goal and that students can learn strategies for problem-solving from it.

The argument was brought to a conclusion by the suggestion that heuristic and rote methods were not alternatives - a child may build up the multiplication table by experiment, and then learn it!

11 THE EFFECTS OF CALCULATORS AND COMPUTERS ON MATHEMATICS EDUCATION IN THE UK

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Introduction

This paper begins with some remarks on the availability of equipment ('hardware') followed by three general sections: on new ideas for mathematics teaching suggested by the calculator/computer, on the problem of developing these ideas into a usable form ('software'), and on the methods of spreading these developments within the education system ('dissemination').

Next there is a more specific survey of recent developments in this field, including a note on public examinations, and finally a reference list of sources of further information and some details of hardware costs and availability. Much of the discussion will relate to calculators rather than computers; the former have been fairly widely available for some five or six years, while microcomputers have begun to appear in schools for the most part only during the past two years.

This paper will restrict itself to mathematics in relation to calculators and computers - we shall not be concerned with the training of computer personnel or of computer scientists, nor with what in the UK is called 'computer education' or 'computer studies' (a new and somewhat ill-defined 'subject' dealing with the history, structure, application and social implications of computers). This almost always includes some practical experience of programming and in some cases serves as an introduction to more specialised computer science courses at 16-18 or post-18 levels. Nor do we consider here the uses of computers as educational aids (CAL, CAI) except insofar as they relate specifically to mathematics. (I make just two exceptions to this in what follows, for the computer-related topics of 'algorithm' and 'simulation' seem to be matters of such general educational importance as to merit some discussion in this context.)

The availability of equipment

In any discussion of the effect of calculators and computers on the mathematics curriculum it is difficult not to be over-influenced by the availability or otherwise of facilities which are both cheap and powerful. In 1971 a discussion such as this would have been highly speculative; in 1961 it would have been visionary! In one sense, nothing has changed - many of the ideas were there ten or even twenty years ago - yet, in another sense, everything has changed, because these ideas now become practicable and people take a more serious interest in them. If the hardware were not available, we would not be engaging in this discussion! Less obviously, until we use the equipment it is not always easy, or even possible, to visualise its full potential. As an example, the situation of a child with a calculator has a 'momentum' of its own. The Leicester/Shell Centre project did not start with a preconceived total strategy for calculator use - instead, very sensibly, in some of the early work, children were given calculators with freedom to experiment, and their actions were observed. In such situations new possibilities arise spontaneously, which might not have been anticipated and which would not have arisen in a more structured situation. A third way in which availability of equipment is significant relates to the issue of credibility. It is of little use to explain to teachers what they could do if they had various items of 'exotic' equipment - they may be more concerned about mundane matters such as the adequacy of the supply of exercise books in the school, and dismiss these new ideas as pie in the sky or yet another education 'fad'.

At present two contradictory statements are simulaneously true (I realise that is an astonishing statement to make to a mathematical gathering!). On the one hand there are teachers who would like to use computers or calculators but who cannot obtain the equipment they need. On the other hand, teachers already have available powerful equipment which they are unable to use fully in teaching, since they do not know how. This apparent 'paradox' is easily resolved: it lies partly in the fact that two different groups of teachers are being considered, and partly in that few of us are capable of seeing all the possibilities inherent in the equipment we have. The first statement is concerned with issues of availability of equipment and its cost, and with compatibility between different items. ('This program will run on an Apple - there is no version yet available for the PET or 380Z.' 'It would be helpful to have printed output as well as that on the VDU screen but a printer and interface unit will be too expensive.' 'Our school has 1,000 pupils and one microcomputer).' The other side of the picture is seen in such comments as 'How can I use our single microcomputer in teaching?' 'What programs are there available?', 'Do I have to write them myself?', or even, 'Many of the children in my class have their own calculators they bring them to school but I won't let them use them in maths lesson.'

Yet, having raised this important issue of equipment, I want now to abandon it! It would be wrong to focus too much attention on 'hardware'. To do so leads to distortions like those of trying to fit an 8K program into a 4K machine, or an undue preoccupation with programming efficiency to save machine time (remember that human time is perhaps more valuable!). These may be interesting intellectual challenges, but they lead to solutions which are rapidly overtaken by improved technology, as memory becomes cheaper, operating speeds increase, etc. I call them distortions because they obscure the major considerations - the ideas, and it is to these that I now wish to turn.

Some ideas for a calculator/computer oriented mathematics curriculum

I shall try to adopt the pattern of illustrating each application by one or more examples, and then give references to other sources of similar material. Clearly there can be no true dividing line between calculators and computers, for already there exist hand-held microcomputers which are capable of executing simple BASIC programs, with a one-line display for program lines or output. However, in what follows a calculator will be taken to mean a device costing perhaps £20 at most, and a microcomputer one costing £500 to £1,000.

In the long term, calculating devices are bound to alter curriculum content. New material will be included or given greater emphasis (iterative methods, discussion of accuracy, 'realistic' data, especially in statistics, matrix multiplication and inversion); some material will be discarded (logs?, 'grouping' and 'coding' for statistical calculations?). Perhaps of greater immediate significance, what is retained will be modified, and this will be the major emphasis in the following remarks.

Uses of calculators

(Editor's note: many of the general references in this section appear in Bell, 1978, Blakeley, 1980, Kaner, 1980

1. Introducing or even 'provoking' new concepts: the sequence 300 → 30 → 3 → 0.3 ... (successive division by 10) for place-value; a child suggests $15^2 = {=}4 \times 16$ - the ensuing discussion could lead to $n^2 - 1 = (n + 1)(n + 1)$ negative numbers and decimals arise unprompted; children may be given a more 'tangible' approach to square roots (Kaner, 1980; Bell, 1978; Watson, 1981).

2. Concept reinforcement: $93 \times 8\boxed{} = 8\boxed{}\boxed{}1$ (Johnson, 1981); 'forecast and check', practice via games - 'unarmed combat' (mind versus calculator) (Watson, 1980), estimation (Tyler, 1981).

3. Exposing misunderstandings (see eg $15^2 = 14 \times 16$ above); allowing concentration on strategy rather than details (eg compound interest, Kaner, 1980). (The CSMS research at Chelsea showed that many difficulties met by children are related to whether to multiply, rather than how to multiply (Hart, 1981).)

4. Providing realism: going beyond the 3, 4, 5 triangle and circles or radius 7k to the use of realistic data, perhaps collected by pupils themselves - eg in statistics, trigonometry, compound interest and hire purchase; use as a 'tool' eg scientific calculations, enabling modelling exercises to be developed, disregarding the commplexity of the calculations involved. (Blakeley, 1980, quotes an interesting example of the growth of buffalo populations.)

5. Providing motivation; related to the provision of realism, this is particularly relevant for the low-attainer in mathematics. Those of us who believe that 'there is more to mathematics than just arithmetic' will welcome the way in which the calculator 'shrinks' the importance of arithmetic.

6. Easing or supporting work in some areas - eg reinforcing the importance of checking and estimating, while making it less laborious, allowing the examination of bounds of accuracy and the effects of measurement errors, speeding the computation of values for graph-plotting (note, though, that the microcomputer can 'short-circuit' this process even more effectively).

7. Encouraging investigations: of number patterns $(1+3+5+7 = 16 = 4^2)$, of conjectures ($n(n+1)(n+2)(n+3) + 1$ is a perfect square; $\sin 2a = 2 \sin a$?), of the calculator itself (as a 'black box' - 'what will happen if I press this key?'), and encouraging generalisations. (See Bell, 1978; Watson, 1979.)

8. Suggesting new problems, eg 'How can I get 17 in the display using only the keys ③, + ⑤ ? (What is the quickest way? Can I get every number in this way? What is the largest number not expressible as $pa + qb$ for given p, q and a, b positive integers?); why do some interative processes converge and others diverge?: does Simpson's rule give faster convergence than the Trapezium rule?

9. Suggesting new algorithms: eg $72 \times 49 = (70+2) \times (50-1) = \ldots$ (see Jones, 1981); (intelligent) trial and error, eg solve $218+x = 259$ by adding to 218 until we reach 259; solve $x^2 = 31$ by successive trials, 5 (too small), 6 (too big), 5.5 (too big) ... (the method of bisection - also applicable to $x^2+x = 31$, $x^3+x = 31$, and leading to more efficient iterative methods). See Watson, 1980 - some of this work suggests a reappraisal of the role of algebra (Mathematical Association, 1981).

Uses of the computer

Where it is available, the microcomputer can obviously serve as a fast calculator, with two additional advantages: that of graphical output rather than tabulation (eg for plotting sin x + sin 3x; solving differential equations by step-by-step methods; illustrating population growth and competition), and that of quick repetition of calculations (eg of a whole series of iterations), or repeated statistical sampling from a population (which may be combined with graphical display to give histograms of sample statistics).

A few specific examples may be mentioned:

1. drill and practice (the program 'Autofractions' from ITMA);

2. concept introduction (introduction to time/temperature and time/distance graphs - 'Juneday' and 'Eureka' programs from ITMA);

3. simulation - collecting problems, queueing problems, (Watson, 1980) 'experimental' verification of tests involving the Normal distribution, 't' distribution, etc (Holmes, 1975; Watson, 1974); and as a 'calculating aid' for simulations in the sciences (Boyle's Law, gravitational attraction, inheritance, transport networks - Schools Council Computers across the Curriculum Project, 1978);

4. geometry - illustration of transformations - rotations, reflections, enlargement .....; 'Turtle' geometry (Papert, 1972);

5. study of algorithms - combination and modification of algorithms (see Papert, 1972, 1981, Engel, 1979.)

(General references for the above are Holmes, 1975, Watson, 1980, Schools Council Computers across the Curriculum Project, 1978.)

So far we have been considering ways in which the calculator/computer can be used in the mathematics curriculum, taking for granted that this is a desirable direction in which to move. I personally have no doubt of this, yet it must be stated there has been opposition, some of it vociferous, in particular to the use of calculators in schools. This seems now to be lessening as they become more widely available; there may be some feeling of 'accepting the inevitable - even if unpleasant'. But it is fair to say that perhaps a majority of primary school (5-11) teachers at this time would see no place for the calculator in their work.

The objections sometimes voiced on technical grounds ('What will they do when the calculator breaks down/when the batteries run out?') are becoming less tenable. Others are based on a form of educational 'inertia' ('They ought still to be able to do long multiplication and division by paper and pencil algorithms'), on a concern for accuracy ('How will they know they have got the wrong answers?') or on the realisation that mathematics has to do with the understanding ('We don't just want to become a nation of button pushers'). I believe all these objections can be countered adequately and rationally, and that attitudes to the calculator are gradually becoming more favourable. The process is slow and sometimes hindered by superficial statements in the local and popular press.

Some teachers in secondary schools who are determined not to have anything to do with the calculator are able to produce an unanswerable objection - 'they are not allowed in the CSE examination'. Leaving aside the issue of whether such a prohibition is desirable (see 'Some recent developments' below), this precludes any consideration of how they might be used in earlier parts of the course to encourage learning.

Developing the ideas

It is one thing to say that calculators or computers have much to offer in the study of (say) iterative processes, it is quite another to devise appropriate teaching materials - textbooks or pupil worksheets, suggestions on presentation and content for teachers, and, where computer systems are concerned, appropriate programs with suitable documentation.

For the most part, material produced in the UK for using the calculator or computer in mathematics teaching has been fragmentary rather than integrated, outline rather than detailed, and directed more to teachers than to their pupils. This is not surprising. given that we are still in the early stages of learning how to use these devices in mathematics teaching. (I believe the remark, that 'it took several hundred years to evolve a mathematics curriculum which did not involve the calculator - and that it may take several decades to assimilate this new and powerful tool into the mathematics curriculum' is attributed to Arthur Engel. Whatever its origin it seems very apposite.)

Considerable resources of money and manpower were devoted to the National Development Programme, and to the Schools Council Computers across the Curriculum Project; their output was by no means insignificant, yet perhaps one of the main outcomes was the realisation that such work is neither simple nor swift - production and testing of support and explanatory material is a larger task than many originally envisaged. This is particularly true of computer-related developments; writing a working computer program is one of the least of our worries.

Some recent developments which have begun to map this largely unexplored territory will be considered later but I would like conclude this section by referring to a recurrent dilemma of curriculum development - the nature and extent of the guidance given to teachers in using the new material. This old problem is not peculiar to our topic. Some developers have provided teachers' notes, leaving the teacher (who 'knows best') to adapt the ideas to pupils' needs; others have produced pupil materials, either assuming that their purpose will be self-evident, or leaving the teacher to make what use of them is felt appropriate. There have been projects which have sought to make their output 'teacher-proof' (usually with total lack of success!) while there are teachers who prefer only an outline, skeletal indication of the ideas, allowing them to follow them up in whatever way seems best, or through what emerges as the lessons proceed. Clearly these 'customers' cannot all be satisfied simultaneously, and since the detailed outworking of these ideas is so time-consuming it may be tempting to adopt the 'outline' approach - justifying this as 'not wanting to spoon-feed the teacher unduly'! This issue is closely related to that of disseminating ideas - for these must be specified in some way if they are to be transmitted - and it is to this that I now turn.

Spreading the ideas

The educational systems of our two countries, though different in several respects, share many common problems, not the least of which is the difficulty of getting new ideas into the system. I will restrict myself to providing some relevant background, though first mentioning two aspects which in their different ways make calculators/computers something of a 'special case'. In the first place the additional obstacle of the cost of equipment (eg sets of calculators for class use, purchase of microcomputers, etc) as well as the cost of new support materials, allied to the technical complexities and problems of incompatibility between equipment of different types, tends to retard progress in this area. In the second place, there is an opposite influence tending to accelerate change - the pervasiveness of calculators and computers in everyday life. There is widespread popular awareness of their presence (the Government is even prepared, under appropriate safeguards, to assist schools to purchase microcomputers) and many children now have access to calculators. (I know of no reliable figures - clearly there are variations from area to area, and with the age of pupils. Some teachers claim that few of their pupils have calculators; others that there are enough among the children for them to share one between two. The provision of school sets of calculators does not seem to be widespread.) The consequence of calculator availability, however is that teachers are not free to ignore the calculator, as they were to ignore 'modern mathematics' when it first appeared. The demands of the syllabus may require them still to teach logarithmic computations, but they will find it increasingly difficult to justify this to pupils who possess calculators and see them used in science lessons, by parents, in shops - in fact, everywhere except in the mathematics classroom!

It may be helpful to mention here some items of background information which have relevance to problems of dissemination.

It is appropriate to remark first that 'computer studies' has developed, at least to some extent, as a 'subject' of the curriculum. In the early days (1964-9) the emphases were generally either oriented towards physics, ('gates', logic circuits, construction of model computers) or to mathematics (programming in high-level languages such as Fortran or Algol, usually with 16- to 18-year-old pupils). There followed a determined attempt to disengage computing from mathematics, and from electronics (the contrast between issues of the journal 'Computer education' at these different periods is interesting). Geographers, economists, biologists, even historians were encouraged to interest themselves in the new subject, and computer studies syllabuses reflected this emphasis.

The Schools Council Computers across the Curriculum Project (SCCC) did not involve mathematics as one of its five subject areas. Yet in many (most?) schools, computer studies was still taught by mathematicians, or ex-mathematicians, and in some cases, was even timetabled at the expense of mathematics. The pendulum has perhaps begun now to swing back - SCCC has now widened its orbit to encompass mathematics, and, indeed, it has been suggested that some of our current problems of shortage of mathematics teachers have been worsened by the diversion of mathematics teachers into computer studies. (It is an interesting comparison that in Scotland computer studies did not emerge as separate, but rather formed part of existing courses in mathematics, business studies, and so on.)

Informal dissemination of ideas about calculators and computers has taken place through the activities of the two teachers' associations (ATM and MA), through their local meetings, at national conferences (both associations have had seminar groups and workshop sessions on calculators/computers for several years now), through their occasional publications, and not least through their journals.

National courses arranged by the Department of Education and Science have provided another opportunity for dissemination, and a variety of other national and local courses have taken place on local initiatives. (Though these are too numerous for all to be known to the writer, they hardly begin the task of encouraging mathematics teachers to reappraise their work in the light of these developments. Yet, in my view, these informal and often small-scale approaches to dissemination are what we should most strive to encourage; regenerating the teaching force is not accomplished 'from the outside in'.) Less well known, but probably of equal significance, are the meetings of the Schools Calculator Working Party associated with the Shell Centre at the University of Nottingham, which has acted as a forum for the exchange of ideas among many of those active in developing work with calculators. This group as such has not produced material - though many of its members have done so as individuals - it has, however, arranged a number of dissemination conferences for advisers, teacher trainers and teachers' centre leaders for use in subsequent dissemination activities, and produced a 'kit' entitled 'How to run a calculator conference'.

A notable characteristic of the School Mathematics Project has been its provision of inservice training and support for teachers following its courses, and one of its recently-published 'In-service training kits' is on calculators.

Some recent developments in the use of calculators

1. The School Mathematics Project has produced a series of booklets based on trial work in schools, and later published by CUP (1979). Some of these seek to supplement the original series of textbooks (Books X, Y, Z), while others are on 'calculator topics'. The main course series (Books 1-5) are now being revised, and calculator applications will be integrated with the work 'as appropriate'.

2. Durham/Schools Council Project. An experiment during 1973-6 in five secondary schools in County Durham was followed by the setting up of a Schools Council Project on Electronic Calculators in the Secondary School. Eleven schools and pupils of all abilities in the 11-16 age range were involved in the pilot stage of material production, and a further fourteen schools in the trial and evaluation stage. This project has just completed its work, and the material produced will be published shortly. Much of this is concerned with the help which calculators can give when familiar mathematical concepts and topics are introduced. The output will be some forty pieces of pupil material with teacher's notes, and a brief explanatory booklet on how the material may be linked into existing courses. Examples of the topics covered are circumference and area of a circle, Pythagoras, sine and cosine rules, iterative method for square roots, conversion between metric and imperial units, electricity bills, an investigation of word lengths.

3. Leicestershire/Shell Centre Project. An account of this project (Bell et al, 1978) was published by the Shell Centre at Nottingham University. The initial work was done in a single primary school in Leicestershire with children in the 5-11 age range. Initially the approach was informal and unstructured, eg children might be given calculators without any explicit instructions as to what use to make of them. Their questions and discoveries were often the starting point for mathematical explorations of a fairly open character. At a later stage, pupils' worksheets have been produced - these retain some of the encouragement to exploration which characterised earlier work, and are obtainable from the Shell Centre. They are intended for 7- to 11-year-olds and have been tried out in a number of Leicestershire schools and elsewhere.

4. Mathematical Association. A subcommittee has produced a booklet 'Calculators have come' (1976) (a series of short articles of suggestions for use) and two more recent pamphlets 'A handbook of number investigations' and 'Algebra with a calculator' (1981). The investigations are ones in which a calculator is often useful as an aid to exploration. The other pamphlet attempts to reassess the place of algebra in the calculator age. (When the content of elementary algebra is examined, it may be seen that much of it is concerned with simplifying and manipulating expressions so as to ease some subsequent arithmetical computation, prompting the question 'Do we any longer need algebra?' It is perhaps reassuring that the authors answer the question with 'Yes, we do'!)

5. The work of the Association of Teachers of Mathematics has already been referred to in the section on dissemination, and almost all of the recent issues of its journal 'Mathematics teaching' contain articles of reviews on the subject of calculators. The editor, D S Fielker, has himself carried out and reported fairly considerable exploratory work with children of 10-12 years. Some other publications which may be of interest are those by the Leapfrogs group (1977), Neill (1978) and Watson (1979, 1981).

Some recent developments in the use of computers

1. The Mathematical Association produced an early and useful pamphlet on 'Computers and the teaching of numerical mathematics in the upper secondary school ' (Ellis, 1971).

2. School Mathematics Project. Some ideas from computing, such as binary arithmetic flowcharting and simple programming, found a place in the SMP syllabus (as in those of other parallel 'modern' projects such as MEI, MME, and SMG) from the early stages. Similarly, topics such as iterative methods, numerical integration, and step-by-step solution of differential equations were incorporated for older pupils. At first, little could be done by way of computer illustration of these topics, though that situation is rapidly changing and SMP schools have been active in the use of the computer in mathematics. COMPACK - a Computer-Oriented Mathematics Package, produced in 1976, comprises three sets of work cards, for programming in BASIC.

3. ITMA (Investigations in Teaching using Microcomputers as an Aid). Funded by a grant from the Department of Industry, this project has ranged fairly widely over the age-ranges and over curriculum subjects though many of its topics are mathematics-related. There are programs and teaching material which introduce such topics as co-ordinates, distance-time graphs, 'function machines' and graph sketching (at the 16-18 age-level). The programs have been developed on the 380Z micro and will shortly be available on disk or cassette-tape, with teaching materials, from Longman.

4. Schools Council Computers across the Curriculum Project. This grew out of the earlier Chelsea Science Simulation Project, and extended that earlier work in biology, physics and chemistry, as well as developing further material in geography and economics. It fits none too neatly into this review since mathematics has only recently been added to its work, and the group concerned, based at the University of Birmingham, has not yet had time to bring material to the stage of publication.

The SCCC project is of interest, however, since it faced problems which will be common to any attempt to promote the widespread use of computers in the teaching of mathematics. Its programs are written in BASIC, and most are aimed at the 16-18 year age group. Considerable effort was devoted to making the programs 'transportable'. They are available free of charge (in versions to suit a variety of mainframe or microcomputers) to purchasers of the (somewhat expensive) teachers' and pupils' materials. These are among the most comprehensive teaching materials yet produced in this country for use alongside CAL programs. Already, however, their 'mainframe' origins are seen as restrictive; for example, they do not provide the more sophisticated high resolution graphics output which is available on present day microcomputers, and a current project is rewriting them to take advantage of these facilities. (Since graphics facilities on microcomputers show little standardisation, the problems of portability seem likely to be especially severe. Experience has shown that even coping with the slight variations in machine-implementation of the widely-used language BASIC causes quite considerable problems. In this respect, therefore, the future outlook can only be described as gloomy, and a lot of adaptation and rewriting is going to be needed if development work done in a particular context is to be made widely available.)

5. HCMMP: The Hertfordshire Computer Managed Mathematics Project. Hertfordshire is an area of the UK where there has been much activity in schools computing. HCMMP began in 1973, aimed at mixed-ability mathematics groups in the 11-13 range: initially in eight schools, it is not firmly established in the country.

The computer is employed in the management of instruction, so that this application is not specific to mathematics, and the pupils need hardly be aware of its involvement. Some of the pupils' worksheets are marked by the teacher; others of a more routine nature, and some which are diagnostic, are marked by the computer (via a mark-sense document reader) which keeps records of pupil tasks and performance, suggests the next assignment for the pupil, etc (and in other similar projects may actually print out an appropriate set of questions for the pupil's next worksheet).

6. Edinburgh University. The Department of Artificial Intelligence has had a long involvement in educational experimentation. Two recent projects will be mentioned here. Du Boulay (1981) reports work with student-teachers who were training to teach pupils in the 5-12 age group. These intending teachers had been identified as being weak in mathematical background so that the work was 'remedial' both in relation to the knowledge and the attitudes of the students. The programming language LOGO (derived from the work of Papert at MIT) was used to teach the students simple programming skills and also to introduce an alternative approach to the topic of fractions, using the graphics capability of the computer. Another experiment, conducted by Dr Peter Ross, began with a pilot study of a small group of 10- to 12-year-olds who were taught LOGO during one year, and in the following year used it to explore a variety of mathematical topics. The main study, currently being carried out in a secondary school, is with the 12-year-old group. Half of them are following their normal mathematics programme; the other half, like their teachers, are being taught LOGO using a specially developed implementation of the language (six microcomputers are in use shared between a class of thirty children). LOGO-related mathematics material (for example, on factors) is being developed to provide illustration and enrichment of the normal mathematics course, which is not being modified in any way for the experiment. Though no detailed results are yet available, this seems a particularly interesting approach.

Further notes on computer activities

The following information, though not centrally concerned with the teaching of mathematics in schools, may serve as useful background; some of the sources mentioned have connections in varying degrees with our theme.

1. The Government-sponsored Microelectronics programme (MEP) began in October 1980 with a £9 million budget; the aim is to spread awareness and use of the microprocessor in schools. Both the electronics/control and programming aspects are involved, and the MEP may be expected to have some impact on mathematics teaching. So far the main activity has been in providing training courses for teachers and in planning the future development of the project. One issue which MEP clearly must consider is how to set up effective mechanisms for software development and exchange - difficult though this will be - otherwise teachers will be unnecessarily duplicating work already done elsewhere ('reinventing the wheel'). We need something comparable to the invention of printing, which ensured that words, once written, could rapidly be made 'public'. Program exchange, via an information network linked to a main frame, may prove to be the solution (cf current practice in Hertfordshire).

2. Societies and journals. The Computer Education Group of the British Computer Society publishes 'Computer education in schools,' which has occasional articles on mathematics teaching; more recently MUSE (Mini- and Microcomputer Users in Secondary Education) has appeared on the scene with its own journal 'Computers in schools'. Other new journals appear almost by the minute: recent additions include 'Computers and education' (Pergamon) and 'Educational computing'. Both CEG and MUSE hold annual conferences.

3. BBC. A new BBC television series (the Computer Literacy Project), linked with a specially commissioned microcomputer is due to start in 1982.

4. Simulation. The idea of computer simulation unites several curriculum strands, from probability, mathematical modelling and computer programming, and seems worth mentioning here for its general educational importance as well as its significance for mathematics. Some relevant references are Open University (1975), Burkhardt (1980) and Watson (1980). See also the 'Journal of mathematical modelling for teachers' (c/o Cranfield Institute of Technology, Bedford MK43 0AL).

A note on examinations

According to many curriculum theorists, the purpose of examination is to reflect the nature of the course and to assess the extent to which its goals have been attained. In practice, examinations tend to act as a controlling agent in the system, shaping the reactions of pupils, teachers, parents, and Local Education Authority officials. Thus it is important in any curriculum change to consider how it will interact with the system of examinations. For us, this raises two major issues:

1. How is the content of mathematics altered by calculator/computer ideas? There is some evidence, especially in Advanced level syllabuses, of ideas which have gained prominence through the advent of computers (iterative methods, flow charts, etc); the SMP and MEI syllabuses provide examples.

The more fundamental restructuring, which I have suggested as likely to arise (see Blakeley, 1980) has not yet *begun* to emerge - it may be some time, even, before logarithmic calculation is accepted as an anachronism.

Examinations in *computer studies* are a separate issue, although some examinations containing elements of both mathematics and computer science have been devised.

2. A second question is the extent to which *calculators* are permitted in public examinations. The detailed picture is confused, and changing, but some general indications may be given:

In the sciences and similar subjects, calculators are *generally* permitted as necessary.

In mathematics there are four varieties of response:

(i) calculators are *not* permitted;
(ii) calculators are not permitted on *certain* maths papers, but allowed in others;
(iii) calculators are allowed: 'questions will be framed so as to confer no advantage on candidates with calculators';
(iv) two *alternative* papers are set, on one of which calculators may *not* be used, on the other they *must* be used. Candidates opt for the type of paper of their choice. (This is the approach used by SMP - about 20% of the SMP candidates (or their schools) now take the 'calculator' option.)

The GCE boards (Ordinary level and Advanced level) generally adopt (ii) or (iii); of the CSE boards, most adopt (i), and a few (three out of thirteen) adopt (ii) or (iii), with the tendency of change being away from (i). Since the examination arrangements are currently being restructured to produce a common examination structure at 16+, we await with interest (and for my part, with some concern) the decisions which will be made with regard to the use of calculators.

Some notes on equipment

Calculators

The first hand-held calculator appeared in the UK in October 1971 at £80; it was a 'four-function' machine (ie +, -, x, ÷). A similar, or better, machine is now available in chain-stores for £3.99.

Calculator sales peaked in Autumn 1978 and are now running at about half that volume, with a reduced variety of models and manufacturers (of which Casio, Hewlett-Packard, and Texas Instruments are the largest).

The TI 30LCD (£10) has an eight-digit display (working to eleven digits), multiple parentheses, battery life of 'several years' (automatic switch-off if neglected for ten minutes); functions include e^x, 10^x, ln^x, logx, y^x, sin, cos, tan and inverse.

The TI 50 (£100) has 100 memory locations, 960 program steps, 170 functions, and stores programs on magnetic card.

The TRS80 (c £100) has a one-line LCD display, is programmable in BASIC with nimety-five numeric variables, and seventy string variables (strings of up to seven characters), and so is a hand-held microcomputer (it can be interfaced to a cassette tape recorder).

Microcomputers

Apple16K version £700 (48K c£800) + monitor (£120).
Pet £450 - comes with 'built-in' monitor

RML 380Z is a more 'expensive' system but is widely used in several influential centres in the UK (Hertfordshire, Birmingham, London, ITMA project).

Sinclair ZX81 £60 (1K memory, expandable to 16K for additional £50) - uses a TV screen as monitor and is a popular 'home computer', as is the Acorn 'Atom' (£120).

Languages

BASIC is very widely used. Pascal has its advocates, and is available on the Apple and some other micros. LOGO is also a possible contender - micro-implementations are being developed at present.

Sources of further information

BBC Computer Literacy Project, Continuing Education, Woodhouse Lane, Leeds LS2 9PX.

Birmingham Educational Computing Centre, Bordesley Centre, Camp Hill, Stratford Road, Birmingham B11 1AR.

Computer Education Group (CEG), c/o N Staffordshire Polytechnic, Beaconside, Stafford.

Educational Computing, 30 Islington Green, London N1 8BJ.

Hertfordshire Advisory Unit for Computer Based Education, Endymion Road, Hatfield, HL1 8AV.

ICL/CES Newsletter, 60 Portman Road, London, RF3 1NR.

ITMA Project, College of St Mark and St John, Derriford Road, Plymouth, PL6 8BH.

Microelectronics Education Project (MEP), Cheviot House, Coach Lane Campus, Newcastle Polytechnic, Newcastle-on-Tyne

Mini- and Microcomputer Users in School Education (MUSE), c/o Ilmington School, Weoley Castle, Birmingham.

References on calculators in schools

See 'A short list of useful references, produced by the Mathematical Association, Teaching Committee (Calculators Subcommittee), and published in 'Mathematics in school', September 1981.

Some of the most significant of these references are:

BELL, A W et al. A calculator experiment in a primary school. Shell Centre, University of Nottingham, 1978 (A general survey and account of the investigation, what children did - plenty of starter ideas here.)

BLAKELEY, B H. The calculator mathematics curriculum of the future. IMA bulletin, August-September 1980

ELLIS, L E et al. Computers and the teaching of numerical mathematics in the upper secondary school. Mathematical Association/Bell, 1971
(Useful reference for sixth-form ideas - adaptable for the calculator.)

JOHNSON, D C. Calculator exploration for concept reinforcement. Mathematics teaching, 95, 1981

JONES, C. 72 x 49. Mathematics teaching, 94, 1981

KANER, P. The calculator: an imperative in secondary school. IMA bulletin, August-September 1980

LEAPFROGS GROUP. Calculators. Action book. Cold Harbour, 1977
(A book of thirty activities and investigations for 11-16 years, though useful outside this range.)

NEILL, H. The use of calculators in the sixth form. County of Durham Education Committee, 1978

TYLER, K. Calculators in primary schools. Worksheets. Shell Centre, University of Nottingham, 1981
(Shell/Leicestershire project - for 7- to 11-year-olds.)

WATSON, F R. Exploring numbers. Series A, B, C. Institute of Education, University of Keele, 1979
(Investigations with a calculator - age ranges 10-13, 13-16, 16-18.)

WATSON, F R. Calculators: promising practices in the teaching of mathematics. IMA bulletin, August-September 1980
(Includes current and recent research.)

WATSON, F R. BACpac: basic arithmetic with a calculator. Institute of Education, University of Keele, 1981
(Worksheets for 13- to 15-year-old low attainers.)

References on computers in schools

BURKHARDT, H. Mathematics and the real world. (Draft ed) University of Nottingham, 1980

DU BOULAY, J B H and HOWE, J A M. LOGO building blocks: student teachers using computer-based mathematics apparatus. (Paper presented at CAL 81 Conference, University of Leeds, 1981)

ENGEL, A. The role of algorithms and computers in teaching mathematics. In : New trends in mathematics teaching, IV. Unesco, 1979

HART, K M. Children's understanding of mathematics, 11-16. Murray, 1981

HOLMES, P. Using computers in the teaching of statistics. Mathematical gazette, 59, 1975

HOOPER, R. NDPCAL. Final report of the Director. Council for Educational Technology, 1977

HOOPER, R and TOYE, I, eds. Computer assisted learning in the UK: some case studies. Council for Educational Technology, 1975

MATHEMATICAL ASSOCIATION. A handbook of number investigations. The Association, 1981

OPEN UNIVERSITY. An algorithmic approach to computing. (Course M251) Open University, 1973

OPEN UNIVERSITY. Computing and computers. (Post-experience course PM 951) Open University, 1973

OPEN UNIVERSITY. Simulation and modelling. (Course T841, Unit 6) Open University, 1975

PAPERT, S. Teaching children to be mathematicians and teaching about mathematics. International journal of mathematical education in science and technology, 1972

PAPERT, S. Mindstorms: children, computers and powerful ideas. Harvester Press, 1981

SCHOOLS COUNCIL. Computers across the Curriculum Project materials. Arnold, 1978 (Biology, chemistry, economics, geography, home-heating, physics.)

THWAITES, B. 1984: mathematics - computers? (Presidential address) IMA bulletin, December 1967

WATSON, F R. KSTATS: an on-line introduction to statistical concepts, using the computer language POP-2. (Unpublished paper) University of Keele, 1974

WATSON, F R. Triangular squares. Mathematics teaching, 85, 86, 87, 1978-9

WATSON, F R. Some suggestions for teaching sixth-form mathematics with a computer. Part 1. Institute of Education, University of Keele, 1980

WATSON, F R. A simple introduction to simulation. Institute of Education, University of Keele, 1980

Discussion developed on the place of computer programming - should it be regarded as part of mathematics or as a separate discipline? Should it be included in the 'core' curriculum and taught to all pupils?

A distinction was made between the use of calculators, which each pupil may soon have, and microcomputers which will probably never be as common. It was suggested that it is necessary to begin restructuring the curriculum now to allow for the very rapid spread of calculators and computers.

CLOSING DISCUSSION

At this first meeting between the mathematics educators of the United Kingdom and the Union of Soviet Socialist Republics, a general understanding has been reached of the place of mathematics in the school curriculum of our two countries. Despite differences in the organisation of the two systems of education, a number of common problems and areas of interest have been identified. It is hoped that future meetings can build on this foundation, and provide an opportunity to explore in depth some of these topics. It is thought that the following would repay further study:

1. minimum content and standards of attainment for all pupils;
2. principles underlying curriculum construction and the compiling of textbooks;
3. teaching mathematics as a unified subject;
4. how to prepare teachers to implement change;
5. approaches to the teaching of geometry;
6. the relationship between theory and application in the teaching of mathematics.

The following topics were also identified as being of mutual interest:

1. methods of pedagogical research;
2. teacher education;
3. future trends in mathematical education.

A RETROSPECTIVE VIEW OF THE SEMINAR

Dr T J Fletcher, Seminar Chairman

I wish now to look back at the seminar after an interval of some weeks and, as Chairman, to give a personal record of the strongest impressions which remain, and to put forward a few matters for consideration if, as I very much hope, it is possible to arrange further joint activities in the future. At the moment I am studying a number of the books which I was given at the seminar with the intention of writing to some of the Soviet participants, and I hope that other members of the seminar will also correspond with one another.

Organisation

It seems to me that the size of the conference was about right. Larger teams would enable some aspects of mathematical education to be covered in greater detail, but translation inevitably slows up discussion and this means that optimum size is smaller than it would be if the discussions were monolingual.

I think it wise to aim at a structured agenda as we did this time. It does not matter if the initial structure is lost because matters of common interest emerge and develop a structure of their own; it is important to have something to start with. The list of topics which we identified at the end of the seminar and which were taken back to Moscow should be a good starting point for another conference.

It is very important to prepare papers in advance, to translate them carefully and circulate them in good time. People are often reluctant to write some months ahead, and it is possible that we could have used already existing papers more than we did. Good translation is essential, both at a professional and a social level. We were well served at Oxford and these standards must be maintained.

The visits to schools were important, and I hope that further seminars will also include such visits.

Personal Impressions

The English and Soviet systems present very obvious differences and this is one reason why comparing them is so instructive. It seemed to me many times during the seminar that if the English participants present had been doing the Soviet visitors' jobs they would have acted much as the Soviet visitors did, and vice versa. We found much common ground and recognised that many of the differences between us arose from the very different frameworks within which we operated.

Perhaps the biggest difference, and one which I would want to discuss further, is the Soviet insistence on uniform minimum provision for all - and a minimum which is very high indeed by English standards. Just as the Soviet constitution imposes certain obligations on the educational system so English law insists on the provision of education according to the age, ability and aptitude of the child. It seems to us that we are trying to apply the policy 'to each according to his needs, from each according to his ability' in education (although we may not apply it to our economic life). This is no place in which to continue the discussion, but I am sure that it is a most important issue which is very relevant to policy decisions which we are trying to make in England at the moment.

I would like to compare our attitudes to primary education more than we succeeded in doing at Oxford. I am far from understanding what the USSR is asking of its primary schools.

I would like to discuss mathematics as a logical discipline. The teaching of mathematics in England has many virtues, but logical refinement is certainly not one of them.

I would like to discuss the relation of mathematics to the material world and the implications of this for teaching. Whereas in logic England is demonstrably inferior to most of its neighbours, I think that in the derivation of mathematics from practical experience it may be somewhat better, and in the teaching of applications it is decidedly better! When making this judgement I compare England with most of the countries I know in Western and Eastern Europe; I would like to know better whether to include the USSR.

This leads to the wider issue of the place of mathematics in society. This has utilitarian and cultural arpects. The Cockcroft Report on the teaching of mathematics in schools, with regard to the needs of higher education, employment and adult life generally, will be a most valuable study not only for England and Wales but, I hope, for many other countries as well. It should be published early in 1982 and I would like to discuss it with Soviet friends.

Finally, I would note that in 1984 the International Conference on Mathematical Education will take place at Adelaide, Australia. I do not know if I will be there myself, but I know that many of my English friends will be hoping to meet again with mathematics teachers, teacher trainers and administrators from all over the world. It would be an immense pleasure to find some of our Soviet friends in that number.

T J Fletcher
10 November 1981

Programme

Tuesday, September 8

Informal visit to St Giles' Fair

Wednesday, September 9

0915 Welcome by the Seminar Chairman, Dr T J Fletcher CBE, HMI

0915 **Session 1. Introduction**

The school system of England and Wales
(tape/slide programme) B J WILSON

The Soviet system of education A I SHUSTOV

A formative period in English mathematical education:
1900 - 1914 A G HOWSON

1400 **Session 2. Formative years**

Revision of the content of mathematics education in the
schools of the USSR during the 1960s and 1970s V M MONAKHOV

Formative periods in English mathematical education:
developments in the post-war years D A QUADLING

Thursday, September 10

0915 **Session 3. The current scene**

Inquiring into the teaching of mathematics in schools
W H COCKCROFT

Matters of current concern J HERSEE

Basic publications on mathematics education for the
use of the teacher N A YERMOLAEVA

1400 **Session 4. The primary stage**

The current state and future prospects of mathematics teaching
in primary schools in the USSR A M PYSHKALO and V V FIRSOV

The early primary years (ages 4-9) E ALBANY

2000-2130 BBC Schools Television D ROSEVEARE

Friday, September 11

0915 **Session 5. Primary school visit**

Sandhills Primary School, Headington, Oxford
Headmaster: Mr D J RULE

1400 **Session 6. The middle stage**

Mathematics in the middle years (ages 9-13) A W BELL

The teaching of mathematics in Classes 4-8 in Soviet secondary schools V V FIRSOV

1730 Punting on the Cherwell

Saturday, September 12

0915 **Session 7. Research in mathematics education**

Scientific and methodological research in problems concerning the improvement of mathematical education in the USSR V M MONAKHOV

Research in mathematics education in the UK A J BISHOP

1215 Visit to St John's College

Afternoon free

2000 **Session 12. In-service support for teachers**

The improvement of the qualifications of mathematics teachers in those Soviet schools providing a general education N A YERMOLAEVA

Support for the serving mathematics teacher B J WILSON

Sunday, September 13

1000-1830 Sightseeing tour of London

Evening Reception hosted by the Soviet participants

Monday, September 14

0915 **Session 8. Mathematics for life**

Determination of the mathematics required in employment by young people between 16 and 18 years of age A FITZGERALD

The study of modern applications of mathematics in Soviet secondary schools V M MONAKHOV

1400 **Session 9. The Senior stage**

The current state of development of the mathematics course for senior classes (ages 15-17) in Soviet secondary schools, and future prospects V V FIRSOV

Ages 16-18: preparation for advanced studies J HERSEE

1715-2400 Visit to Stratford-on-Avon. Dinner at Royal Shakespeare Theatre Restaurant before 'A midsummer night's dream'

Tuesday, September 15

1000-1400 **Session 10. Secondary school visit**

Gillott's School, Henley-on-Thames
Headmaster: Mr D H W GRUBB

1400-1600 **Session 11. Calculators and computers**

The effect of calculators and computers on mathematics education in the UK F R WATSON

Demonstration of programs designed for use in school mathematics teaching, on the 380Z microcomputer, by Mr T Easterbrook, Adviser in Mathematics, Oxfordshire LEA

(Session at Gillott's School by kind permission of the Headmaster)

Tea with senior staff of Gillott's School

2000 Seminar Dinner with invited guests at St Antony's College
Host and hostess: Mr and Mrs P B NAYLOR
(Mr Naylor is Controller of the Overseas Division for Europe of The British Council).

Wednesday, September 16

Departure

List of Participants

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